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THESIS

A KALMAN FILTER APPLICATION TO THE
ADVANCED TACTICAL INERTIAL GUIDANCE
SYSTEM OF THE AIR-LAUNCHED LOW VOLUME
RAMJET CRUISE MISSILE

by
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December 1976

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RAMJET CRUISE MISSILE

by

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requirements for the degree of

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ABSTRACT

A Montecarlo simulation is conducted to ascertain performance of the ATIGS system in a proposed air-launched cruise missile configuration. The simulation is conducted within a local-level inertial frame consisting of down-range, cross-range and up as primary reference vectors. Efforts are made to measure the relative effects associated with the intended pure position reset provided by a micrad sensor as compared with those effects which could be expected from a linear suboptimal Kalman filtering scheme used in conjunction with the MICRAD sensor

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I. INTRODUCTION

The purpose of the Advanced Tactical Inertial Guidance System(ATIGS) program is to demonstrate the feasibility of a low cost inertial system to be used in the Air Launched Low Volume Ramjet (ALVRJ) cruise missile for mid course guidance. Within the framework of this stated purpose lies the intent to furnish moderate accuracy in a strapdown inertial navigator with high reliability of operation.

The strapdown inertial system requires a computer to provide inertial reference , hence the possibility of extending the computer's capability by installation of Kalman filtering algorithms is seen as an area for investigation. Previous work (ref. 1,2) in this field indicates that the computational burden associated with the Kalman filter limits its usefulness when position updating systems in the missile give highly accurate measurements of actual position. Most of the aforementioned computational burden resulted from the on-line gain generation required by a non-linear model within the Kalman filter. Hence if a linear model with sufficient performance were to be incorporated and the Kalman gains generated off-line and stored, then possibly the velocity estimation errors which are largely unaffected by the position updates could be reduced.

The purpose of this study was then threefold

- 1) Test a linear model of missile dynamics for use as a simulation tool.

2) Determine the inertial navigator accuracy within the six degree of freedom simulation when a pure position reset device is installed which provides position updates at two points along the flight path.

3) Determine improvements in missile performance if a Kalman filtering scheme were installed to estimate missile states between position updates.

The means by which accomplishment of the desired purposes was obtained were various Montecarlo simulations utilizing existing data on the proposed inertial guidance system. Extensive work, both in testing of physical equipment and in simulation, had previously been accomplished by various departments of the Naval Weapons Center, China Lake, California. Hence accurate data as to component performance were available. These data were utilized to construct models of the components for computer simulation.

The simulation of a strapdown inertial guidance system requires the nonlinear computations relating observed accelerations to inertial frame coordinates. This is normally accomplished within the guidance-navigator algorithm by a suitably chosen set of state variables and their related non-linear dynamics. The essential idea behind the linearization technique used in this report is that the non-linear calculations relating accelerations and angular rates to velocity changes within the inertial frame could be accomplished upon observation of the said accelerations and rates and utilized as forcing functions for a linear model of system dynamics. This corresponds to a free inertial system with observations physically aligned in the inertial frame of reference. Thus velocity changes in the inertial frame of the strapdown guidance system would be the non-linear combination of accelerations, angles and

angle rates which are treated as inputs

The model dynamics then are simple linear equations for which Kalman filtering gains can be calculated and stored.

The proof of the above linearization technique would be in comparison of the existing empirical performance of the ATIGS system with the observed corresponding simulation of the system without Kalman filtering installed. Ref.(3) provides ample data of the drift of the ATIGS system as a function of time under actual flight conditions. The data are for a pod mounted version of ATIGS installed on an A-7 aircraft. Information from this report indicated that ground test drift of the system was on the order of 1 nautical mile (nm) per hour under controlled temperature conditions. Under free flight test conditions without temperature control, performance was degraded to 4 nm per hour. The temperature instability was not incorporated into the simulation due to current effort to provide corrective measures within ATIGS. Hence verification of the model was assumed if simulation indicated drifts of 1 to 2 nm per hour.

Once verification of inertial-physical model was assumed, the next phase of observation of effect of pure position update was commenced. Ref.2 in an unclassified portion, contends that the optimal weighting of filtered position estimates and highly accurate measurement of position is such that the filtered estimates are ignored. This being the case, the computational burden imposed by a time varying Kalman filter may be unwarranted. The implicit assumption here is that the velocity errors incurred in an unfiltered system are not significantly decreased by the filter. Therefore the position reset feature would be sufficient to provide required accuracy at mid-course termination. This conclusion was tested by simulation of

missile flight under conditions of increased noise levels within the ATIGS system and comparison with "normal" performance obtained, which allowed visualization of the magnitude of end point error incurred under conditions of pure position reset and different noise level sensors.

The final phase of study was the filtering of the sensor outputs to provide more accurate estimates of velocity throughout the flight. No attempt was made to filter the position update measurements due to their reported accuracy ($\sigma=50$ ft). Thus any gains in performance would have to be from the filtered estimates of position after position update and continuous filtered estimates of velocity. The primary indicator of accuracy in line with ref.2 was taken to be cross range error and cross range error covariance as a function of time.

II. BASIC DESCRIPTION OF THE ATIGS EQUIPMENT UTILIZATION

The approach selected for implementation of ATIGS within an actual cruise missile, was to employ a high-speed processor to handle transformation updating, earth rate torquing and other minor tasks in order to save on time requirements on the more complex navigation and guidance computer. This peripheral processor would then supply the central processing unit(CPU) with the necessary information that it required to compute position within the inertial guidance frame.

Thus the basic ATIGS unit involves ring laser gyros and accelerometers providing information to the peripheral processor wherein after suitable transformation, inertially referenced changes in state variables are supplied to the main navigation-guidance computer. The main computer (CPU) then has an auxiliary input from an external position measuring device, the Microwave Area Correlation fixer(MICRAD). The MICRAD system is designed to provide highly accurate position measurements at two or three preselected checkpoints along the route of flight. These position updates would then be utilized within the CPU to reset the inertial guidance estimate of position.

At present the only filtering system installed is an application to the initial alignment scheme wherein a two stage initialization process is used to align the missile inertial frame with the parent aircraft inertial frame. Filtering is not used presently during midcourse guidance due to the position checkpoint feature and the short time of flight.

A block diagram indicating proposed ATIGS utilization within the ALVRJ is shown in fig.1.

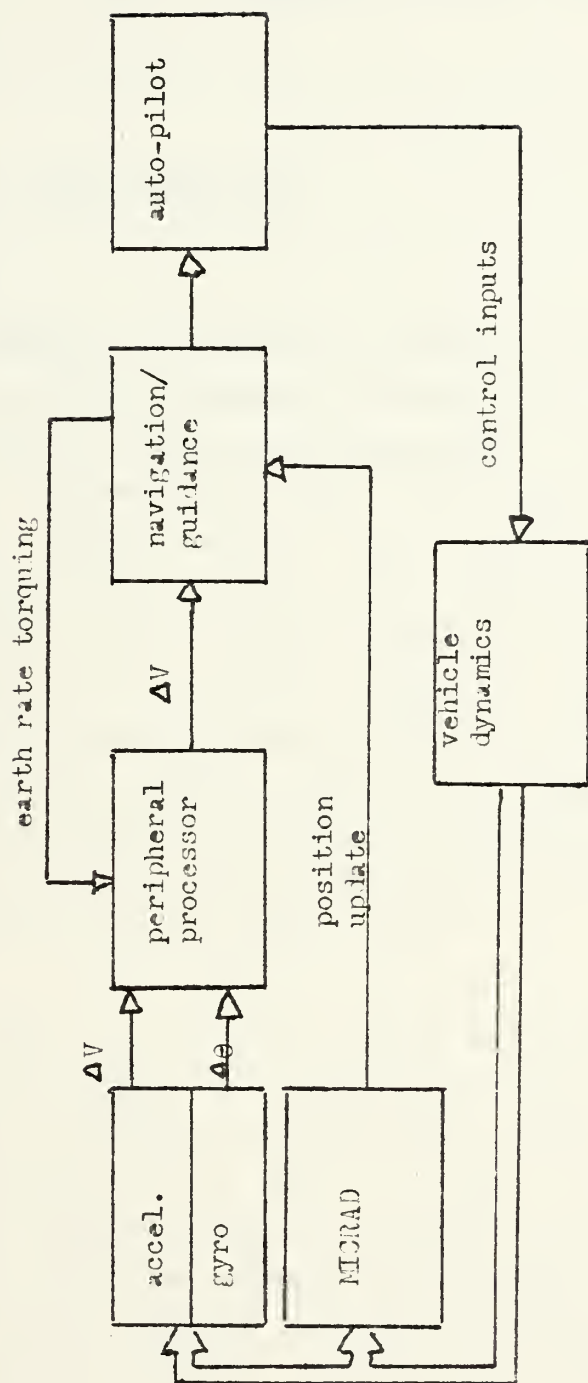


Figure 1 - FUNCTIONAL BLOCK DIAGRAM OF THE ATIGS
INSTALLATION WITHIN THE ALVRJ CRUISE MISSILE

III. INERTIAL SYSTEM SIMULATION

A. SYSTEM CONSIDERATIONS

The inertial navigation system incorporated in ATIGS consists of the Honeywell GG-1300 Ring Laser gyro (RLG) and the Sunstrand Q-flex accelerometer acting as sensors. Empirical performance data for these sensors are found in table (1). Both sensors are considered to be of the integrating type in that the output of the RLG is in the form of total angle change per pulse and the output of the accelerometers are total velocity change per pulse. The measurement is accomplished within the RLG by means of a counter system which totals the number of fringe pattern passages during each pulse period and similarly within the accelerometers, the total velocity change is proportional to the magnitude of the output pulse.

In view of the above characteristics it was felt that the inertial system as diagrammed in fig.4 could be modeled simply and linearly by using the outputs of the sensors as forcing functions vice part of the state vectors. The similarity between fig.1 and fig.4 should be noted. This would result in a net reduction in number of state variables by allowing the missile dynamics to consist of second order equations of displacement and first order equations for angular motion.

B. INERTIAL FRAME OF REFERENCE

1. ALVRJ Implementation

The proposed ATIGS application to the ALVRJ utilises a local-level co-ordinate frame for navigation to the target. The local-level frame is characterized by North, East and up as the respective axis of calculations. The non-spherical nature of the earth introduces an angle calculation which relates the local gravity vector to the position vector of the origin from the earth's center. In the ATIGS unit, the gravity vector calculation is accomplished by an inverse square gravitation model

$$G = -(KM/R^3) R \quad (1)$$

where

G	gravity vector
K	earth's gravitation constant
M	mass of the earth
R	position vector from earth center to vehicle

which approximates the local gravity vector to the desired degree of accuracy.

The vector output of an orthogonal set of accelerometers is the geometric sum of all forces which act upon the vehicle and of course gravity is included. Since the above calculation is dependent on position, then the gravity vector is not constant during the time of flight. Thus to distinguish between the effect of external forces applied to the missile and the change in the gravity vector an equation such as

$$F_a = C_a^i \cdot \dot{R}_i - G \quad (2)$$

F_a force exerted on instruments
 C_a^i coordinate transformation relating inertial
axis(i) to accelerometer axis(a)
 R_i inertially referenced acceleration
 G gravity vector

resolves the time varying accelerometer outputs.

ATIGS accomplishes the above procedure within the missile and calculates the proper direction and range for a direct steer to the target.

2. System Simulation

The simulation of the ATIGS mission began by approximating the local-level inertial frame defined in 1. above as a down-range, cross-range and up frame of reference. Due to the limited range of the missile the gravity vector was considered constant and known, hence the simulation simplified to a simple cartesian co-ordinate space wherein the navigator assumes knowledge of initial position, target position and range to target. The correct heading to the target was assumed to be the positive x-direction with the right-hand system defining positive cross-range accordingly.

The initial position of the inertial frame of reference was taken to be the origin and instantaneous headings were taken to be the difference between the longitudinal body axis and the positive x direction.

The ALVRJ maintains a constant wings level flight and this restriction was also placed upon the simulation.

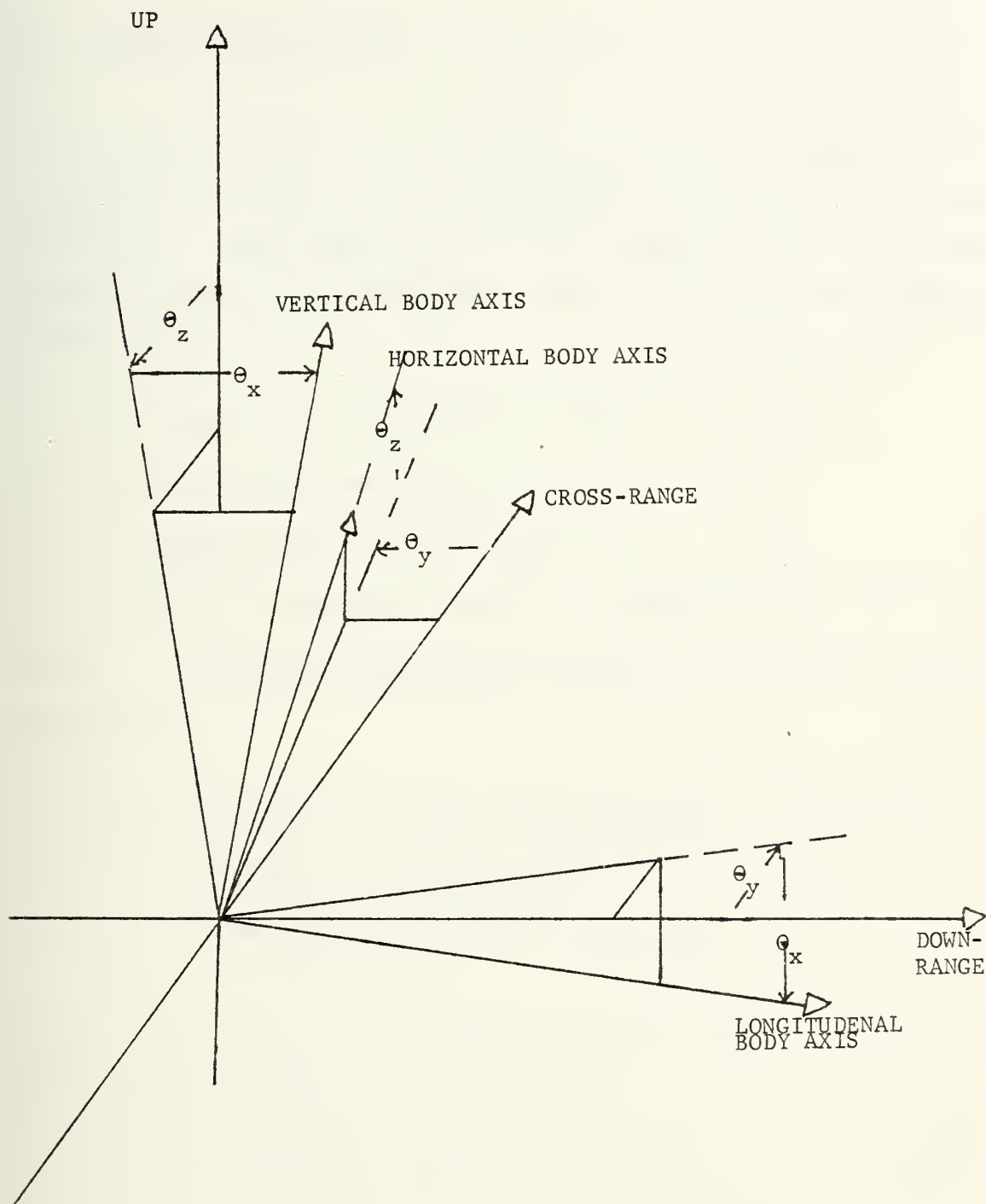


Figure 2 - SIMULATION FRAME OF REFERENCE

C. GENERAL DEVELOPMENT

Defining a set of state variables for an inertial system undergoing arbitrary two dimensional translation and making the careful restriction that small angles and very small angular rates are involved, one obtains (neglecting all noise inputs)

x Distance from origin down-range
 $\dot{x}$ Velocity in down-range direction
 Y Distance from track centerline
 $\dot{Y}$ Velocity component vertical to centerline
 Theta Angular displacement of body longitudinal axis to centerline

Due to the small angles and negligible effect of angular rates one can approximate the accelerometer outputs as

$$\begin{aligned} Z_1 &= \text{accel. in longitudinal body} \\ &\quad \text{axis} = A_{1b} = \Delta V_{1b} \\ Z_2 &= \text{accel. in lateral body axis} = \\ &\quad A_{2b} = \Delta V_{2b} \end{aligned} \quad (3)$$

where the B subscript indicates body axis. The output of the single gyro is

$$Z_3 = \dot{\theta}_1 \Delta T = \Delta \theta_1 \quad (4)$$

Now

$$\begin{aligned} \Delta \dot{X} &= \Delta V_{1b} \cos \theta_1 - \Delta V_{2b} \sin \theta_1 + V_{1b} \Delta \cos \theta_1 \\ &\quad - V_{2b} \Delta \sin \theta_1 \\ \Delta \dot{Y} &= \Delta V_{1b} \sin \theta_1 + \Delta V_{2b} \cos \theta_1 + V_{1b} \Delta \sin \theta_1 \\ &\quad + V_{2b} \Delta \cos \theta_1 \end{aligned} \quad (5)$$

Where

$$\Delta \dot{X} = Z_1 \cos \theta_1 - Z_2 \sin \theta_1 + V_{1b} \Delta \cos \theta_1 - V_{2b} \Delta \sin \theta_1 \quad (6)$$

$$\Delta \dot{Y} = Z_1 \sin \theta_1 + Z_2 \cos \theta_1 + V_{1b} \Delta \sin \theta_1 + V_{2b} \Delta \cos \theta_1$$

Which can be further approximated by

$$\begin{aligned} \Delta \dot{X} &\approx Z_1 - Z_2 \theta_1 - V_{1b} \theta_1 \Delta \theta_1 - V_{2b} \Delta \theta_1 \\ &\approx Z_1 - Z_2 \theta_1 - V_{2b} Z_3 \\ \Delta \dot{Y} &\approx Z_1 \theta_1 + Z_2 + V_{1b} \Delta \theta_1 - V_{2b} \theta_1 \Delta \theta_1 \\ &\approx Z_1 \theta_1 + Z_2 + V_{1b} Z_3 \end{aligned} \quad (7)$$

Further utilization of small angle approximation yields

$$\begin{aligned} V_{1b} &= \dot{X} \\ V_{2b} &= \dot{Y} \end{aligned} \quad (8)$$

Hence

$$\begin{aligned} \Delta \dot{X} &= Z_1 - Z_2 \theta_1 - \dot{Y} Z_3 \\ \Delta \dot{Y} &= Z_1 \theta_1 + Z_2 + \dot{X} Z_3 \end{aligned} \quad (8a)$$

and for unit time intervals the discrete state equations are

$$\begin{aligned} X(k+1) &= X(k) + \dot{X}(k) + .5 * \Delta \dot{X}(k) \\ \dot{X}(k+1) &= \dot{X}(k) + \Delta \dot{X}(k) \\ Y(k+1) &= Y(k) + \dot{Y}(k) + .5 * \Delta \dot{Y}(k) \\ \dot{Y}(k+1) &= \dot{Y}(k) + \Delta \dot{Y}(k) \\ \theta_1(k+1) &= \theta_1(k) + \Delta \theta_1(k) \end{aligned} \quad (9)$$

Thus the observations can be treated as inputs to the system after appropriate substitution. The above model can be expanded to three dimensions and six degrees of freedom by the addition of one cartesian and two angular coordinates which would then consist of

$$Z(k), \dot{Z}(k), \theta_2, \theta_3$$

for a total of 9 states.

This development was accomplished without noise considerations. In the physical system noise would exist in the form of measurement noise in both the accelerometers and the gyros. Hence

$$\begin{aligned} Z_1 &= \Delta V_{1b} + \gamma_1^* \\ Z_2 &= \Delta V_{2b} + \gamma_2^* \\ Z_3 &= \Delta \theta_1 + \varphi_1 \end{aligned} \quad (10)$$

and

$$\begin{aligned} \Delta \dot{X} &= Z_1 - Z_2 \theta_1 - \dot{Y} Z_3 - \gamma_1^* - \gamma_2^* \theta_1 - \dot{Y} \varphi_1 \\ \Delta \dot{Y} &= Z_1 \theta_1 + Z_2 + \dot{X} Z_3 - \gamma_1^* \theta_1 + \gamma_2^* + \dot{X} \varphi_1 \end{aligned} \quad (11)$$

The purpose of this approach was to utilize the outputs of the sensors as forcing functions for the linear model. Hence in the preprocessor the non-linear calculations involving observations and states can easily be accomplished such that one then obtains

$$\begin{aligned} U_1' &= Z_1 - Z_2 \theta_1 - \dot{Y} Z_3 \\ U_2' &= Z_1 \theta_1 + Z_2 + \dot{X} Z_3 \\ U_3' &= Z_3 \end{aligned} \quad (12)$$

as hypothetical and known forcing functions. Thus

substitution into the model of the system of $U_1'(k)$ for $\Delta \dot{x}(k)$ and $U_2'(k)$ for $\Delta \dot{y}(k)$ would result in

$$\begin{aligned}
 X(k+1) &= X(k) + \dot{X}(k) + .5*(\Delta \dot{X}(k) + \delta_1 \\
 &\quad + \delta_2 \theta_1 + \dot{Y}(k) \varphi_1) \\
 \dot{X}(k+1) &= \dot{X}(k) + \Delta \dot{X}(k) + \delta_1' + \delta_2' \theta_1(k) \\
 &\quad + \dot{Y}(k) \varphi_1 \quad (13) \\
 Y(k+1) &= Y(k) + \dot{Y}(k) + .5*(\Delta \dot{Y}(k) + \delta_1' \theta_1(k) \\
 &\quad + \delta_2' + \dot{X}(k) \varphi_1) \\
 \dot{Y}(k+1) &= \dot{Y}(k) + \Delta \dot{Y}(k) + \delta_1' \theta_1(k) + \delta_2' \\
 &\quad + \dot{X}(k) \varphi_1 \\
 \theta_1(k+1) &= \theta_1(k) + \Delta \theta_1(k) + \varphi_1
 \end{aligned}$$

Hence the net result is the addition of a process noise term to the model. Analysis of this noise term proceeds with the systematic elimination of the non-linear term involving δ and Θ . This is easily justified due to the small value of δ and Θ . Thus one is left with down-range process noise involving δ_1' and $\dot{Y} \varphi_1$ and cross-range noise terms involving δ_2' and $\dot{X} \varphi_1$. Clearly the non-linear terms will dominate. The conclusion that logically follows is that the linearization technique utilized above will obviously be accurate for small angles in a manner proportional to the magnitudes of the quantities $\dot{Y} \varphi_1$ and $\dot{X} \varphi_1$. Furthermore it indicates that Kalman filtering will be most effective in the estimation of angle information and of much smaller benefit in the filtering of accelerometer noise.

PARAMETER	UNIT	PERFORMANCE	UNITS
RANDOM WALK ($^{\circ}$ /hr)	GG-1300-RLG	.0075	1 1hr.
BIAS STABILITY ($^{\circ}$ /hr.)	SAME	.009	1
BIAS SENSITIVITY ($^{\circ}$ /hr. of f)	SAME	.00037	
SCALE FACTOR (%)	SAME	.016	
BIAS UNCERTAINTY (ug)	Q-FLEX	79.0	1
SCALE FACTOR (ug/g)	SAME	63	1

TABLE 1-PUBLISHED UNCERTAINTIES OF THE ATIGS
COMPONENTS

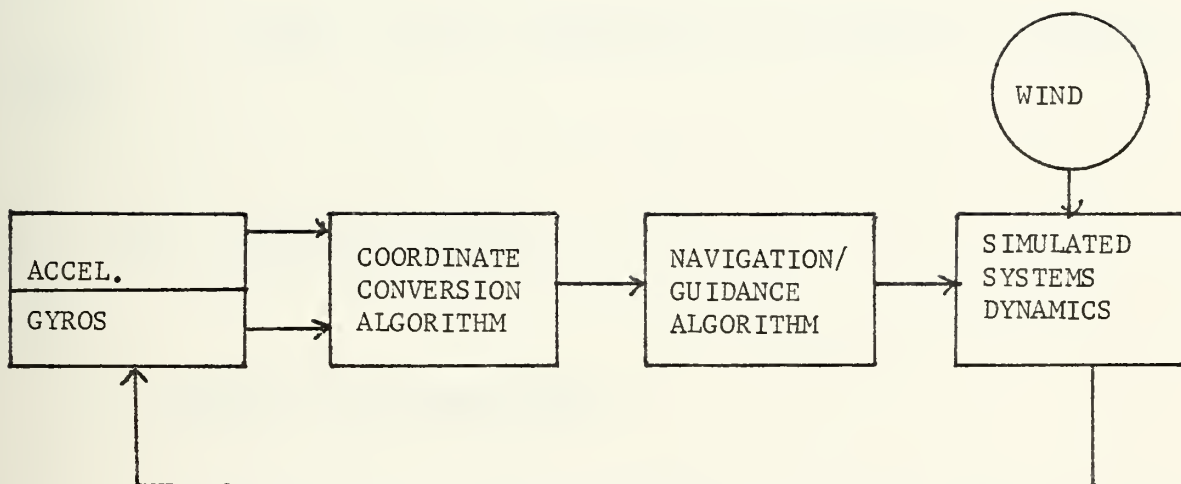


Figure 4 - BLOCK DIAGRAM FOR SIMULATION OF ATIGS
INSTALLATION

Thus the total state vector would consist of 9 states as opposed to the 15 state vector considered essential in reference 2. The above technique was inspired by Kortum in his development (ref. 6) on Kalman filter applications. The inertial computation scheme is based on several assumptions, all of which are results of the short time of flight-short range requirements of the ALVRJ application. These assumptions are

1. Constant gravity vector over a 40 nm. flight path
2. Earth torquing not required
3. Small angle assumptions for largest portions of flight

D. BASIC SIMULATION DESIGN

1. General Considerations

In order to verify that the nine state system would be adequate for modeling purposes, a simulation program to test performance was conceived. The actual missile flight profile includes accelerations after launch

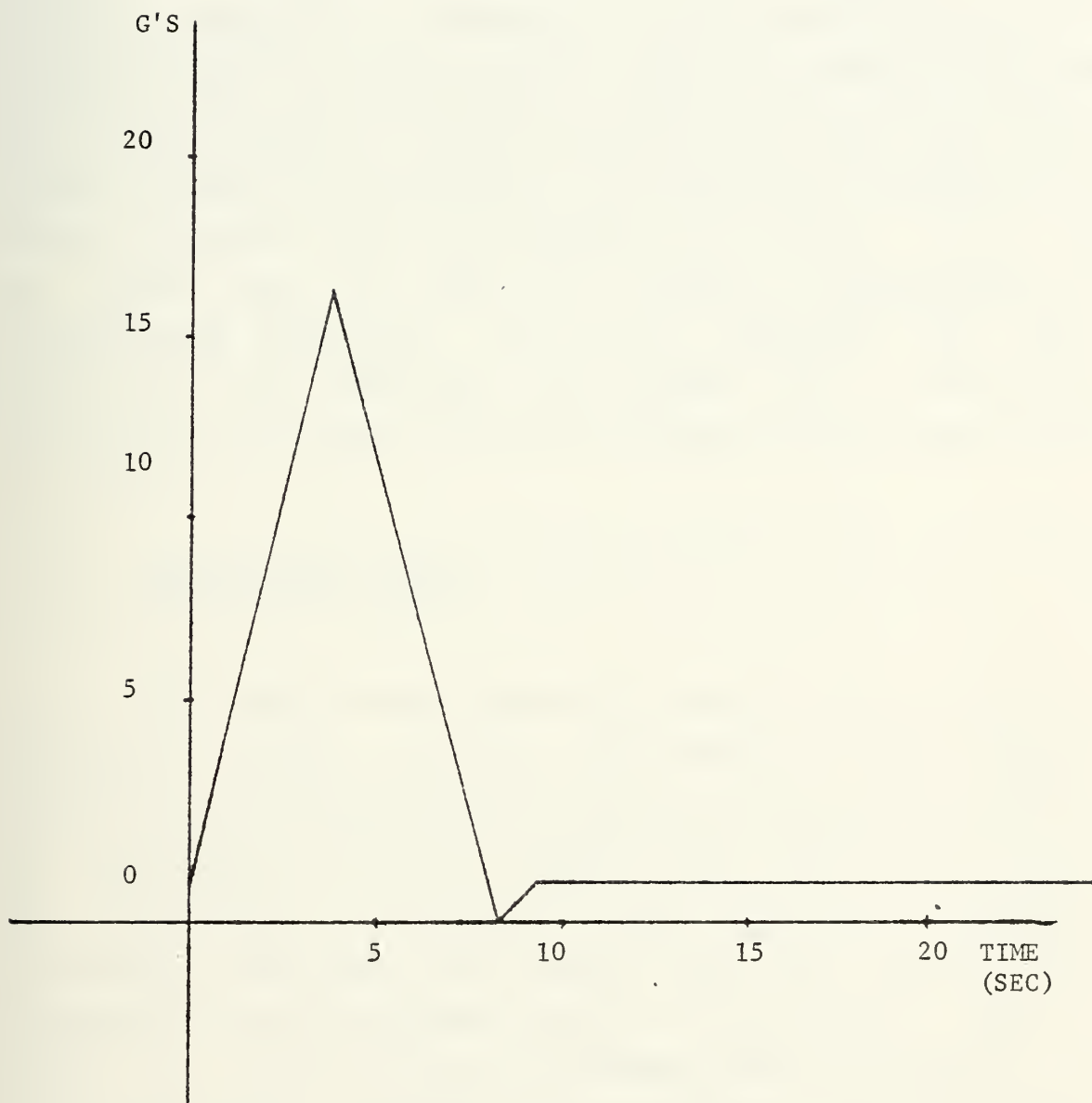


Figure 5 - ACCELERATION PROFILE

up to a maximum of 16 g's(fig.5)from the initial conditions given in fig.6. In addition, it is to be noted that for simplicity a homing type guidance command is given to the missile dynamics. This is recognised as ineffecient in practice but is simple in implementation and provides a basis for comparison purposes. Since missile flight controls respond to airspeed and not inertial speed, the velocities used in the missile dynamics portion of the algorithm are true airspeeds. However the inertial system must always compute in inertial velocities and hence must either adjust its calculations to include this difference or accept any error that this difference may entail. It is to be noted that no means of velocity measurement(i.e. doppler,mach gauge,etc.) is to be provided. In this simulation the difference is ignored due to the large magnitude of inertial velocities obtained and the short time of flight.

2. Noise Input Design

The randcm number generators used in this simulation were of two types; Gaussian and uniform. The Gaussian generators provided the noise inputs to the various sensors and the uniform generators provided the initial conditions and wind effects. The wind effects were such that constant bias directions of positive cross range and negative down-range conditions were imposed. The mean value of wind components in each direction was 30 ft per sec. with a range of ± 8 ft per sec. maximum change per second. No attempt was made to ascertain the relevance of the chosen wind model,its purpose was purely to introduce a bias into the system equations in order that the scale factor noise term of the gyros could be exercised. The scale factor term was finally dropped from the model of the gyros but the wind bias was retained.

The Gaussian generators provided noise inputs to each of the six installed sensors. The chosen model of the accelerometer noise term was

$$\delta_{1,2,3}^* = \text{EOG}_{1,2,3} + \text{WG}_{1,2,3} * A_{1,2,3} \quad (14)$$

where

EOG random bias term held constant over the entire flight but varied prior to each sample in the montecarlo

WG-scale factor term which varies through the flight

The chosen model for the gyro noise term was more complex consisting of bias terms and a random walk term. A random walk generator is described in general terms as

$$\dot{E}_r = u_r \quad (15)$$

where E_r is the error at a given instant and U_r is a white noise term with a standard deviation of σ_{U_r} . The variance of E_r grows linearly with time according to the relation

$$\sigma_{E_r}^2 = t \sigma_{u_r}^2 \quad (16)$$

with σ_{E_r} given empirically in table 1 as random walk in $^\circ/\text{hr}$ with an uncertainty of $\pm 14\% = .0075$. For a sample generator to be used every second

$$\sigma_{u_r} = \sigma_{E_r} / 60 \quad (17)$$

An error from each gyro is introduced into the inertial computation which is treated as a change in Θ .

$$\varphi(t) = \text{EO} + G(t) \quad (18)$$

PHI error of each gyro per interval of time

EO constant bias term per flight

G result of random walk

3. Changes in Heading Resulting in Velocity Changes

The computation of velocity in the inertial frame consists of terms which relate the change in heading to changes in inertial velocities. For small angular rates the changes in velocity in the inertial frames show as inputs i.e. $AWXX, AWXY, AWYY, AWYZ, AWZZ, AWZX$.

$AWXY$ is the change in velocity in the X (down-range) direction due to a change in direction Θ_y . This is expressed in small angle approximations as

$$\begin{bmatrix} AWX \\ AWY \\ AWZ \end{bmatrix} = \begin{bmatrix} 1 & -\Delta\theta_z & \Delta\theta_y \\ \Delta\theta_z & 1 & \Delta\theta_x \\ -\Delta\theta_y & -\Delta\theta_x & 1 \end{bmatrix} \begin{bmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{bmatrix} \quad (19)$$

AWX is the total change in velocity in the x direction due to small angle change. This calculation is performed in the appropriate missile dynamics portion of the simulation where the $\Delta\theta_i$ are the results of commands from the guidance system of the inertial system. In the inertial system these quantities are treated as $\Delta\theta_m$'s or measured changes and all velocity changes are computed based on gyro outputs.

4. Guidance System Design

The specific algorithm for generation of guidance commands was simple due to the heading type control employed. Inherent in the cross-range, down-range reference frame is the knowledge of distance remaining or "time-to-go" for termination of midcourse guidance and initiation of terminal guidance procedures. A parameter that continues to be significant within this project is the small angle, small rate assumption. In the guidance algorithm the one second time intervals chosen for use would require an inordinately long sequencing operation if commands were given in terms of rates. To clarify this statement, rate systems require an initiation and termination command, which for one second intervals would require a two second execution time. Therefore the guidance system employed within this project determines total angle change necessary and then commands an automatic pilot to accomplish this change. Thus the forcing function to the inertial navigator equations is not an input to the rate variables but rather to the angular displacement variables. No process noise was assumed for small angles hence the actual angle change was set equal to the commanded angle change. Notice that this is basically an open-loop process wherein the inertial navigator does not predict the next state based on the commanded heading change but rather on the noisy observed heading change.

The logical question then arises as to the effect of system drift during guidance. Normal procedure would be for a heading change command system in which an error signal generated by the navigator would be driven to null by the rotation of the vehicle. System drift during the heading change operation would result in process noise inputs to the navigator equations. The above was felt to be undesirable due to Kalman filter operation characteristics wherein steady state gains are non-zero for a linear system under process noise. The method of circumventing this discrepancy was to use as the forcing function the observed heading

change for the update of the navigator. Thus in the absence of process noise an accurate indicator of measurement noise would be the difference between observed heading change and commanded heading change. This concept was to be used during the Kalman filter application.

The algorithm for command guidance is given in fig.5. The "time-to-go" concept allows for a continual estimate of distance remaining, and a simple heading calculation computes the heading change necessary for homing.

$$\begin{aligned}\theta(k)_{\text{required}} &= \frac{\text{cross-range position}}{\text{final position} - \text{present position}} \\ &= \frac{Y(k)}{240000 - X(k)}\end{aligned}\quad (20)$$

It should be noted that the flight profile simulation was terminated at a point where the inertially computed down-range position was greater than/or equal to the final position. This would correspond to the switchover point for terminal guidance.

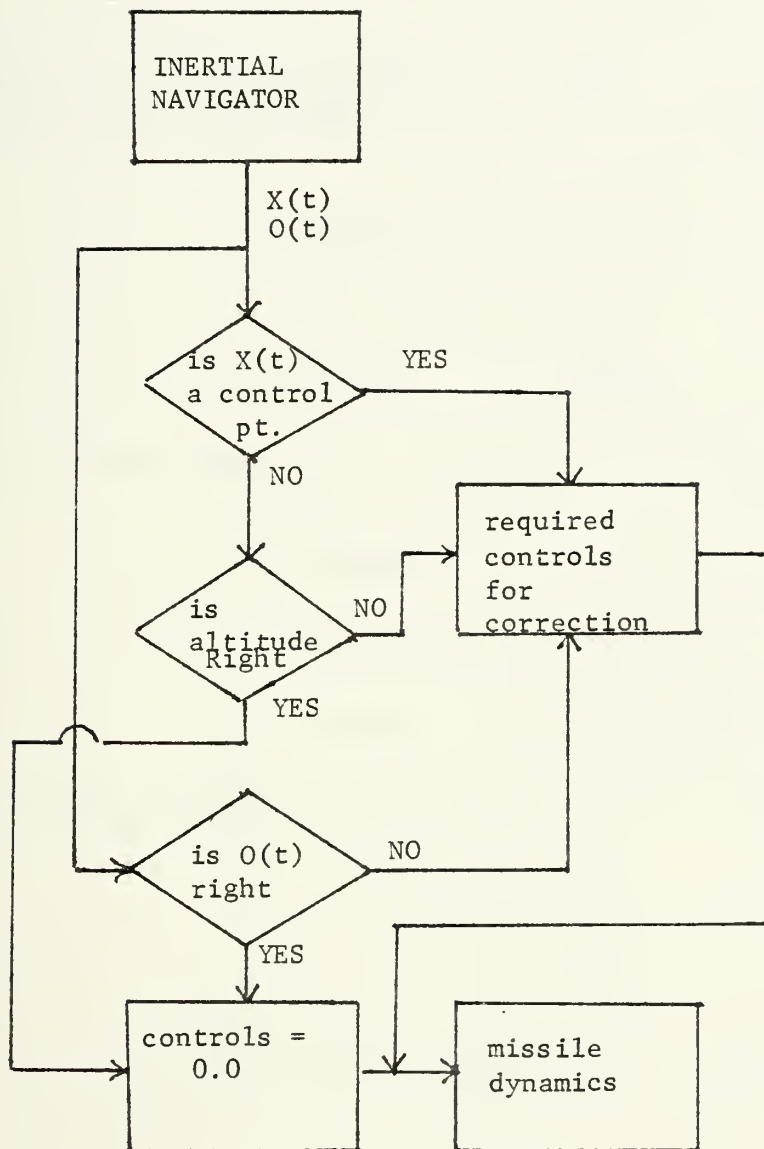


Figure 6 - GUIDANCE COMMANDS ALGORITHM

STATE VARIABLE	COORDINATE	MEAN	STD. DEVIATION
POSITION	downrange	0.0 ft (0.0 m)	1 = 387.0 ft (118 m)
	crossrange	0.0 ft (0.0 m)	1 = 387.0 ft (118 m)
	altitude	35000 ft (10668 m)	1 = 0.0 ft (0.0 m)
VELOCITY	downrange	670 ft/sec (204.2 m/sec)	1 = 6.0 ft/s (1.83 m/s)
	crossrange	670 ft/sec (204.2 m/sec)	1 = 6.0 ft/s (1.83 m/s)
	vertical	0.0 ft /sec (0.0 m/sec)	1 = 0.0 ft/s (0.0 m/s)
ALIGNMENT	θ_1	0.0°	1 = 2 min
	θ_2	0.0°	1 = 2 min
	θ_3	0.0°	1 = 2 min

Figure 7 - INITIAL CONDITIONS AT LAUNCH

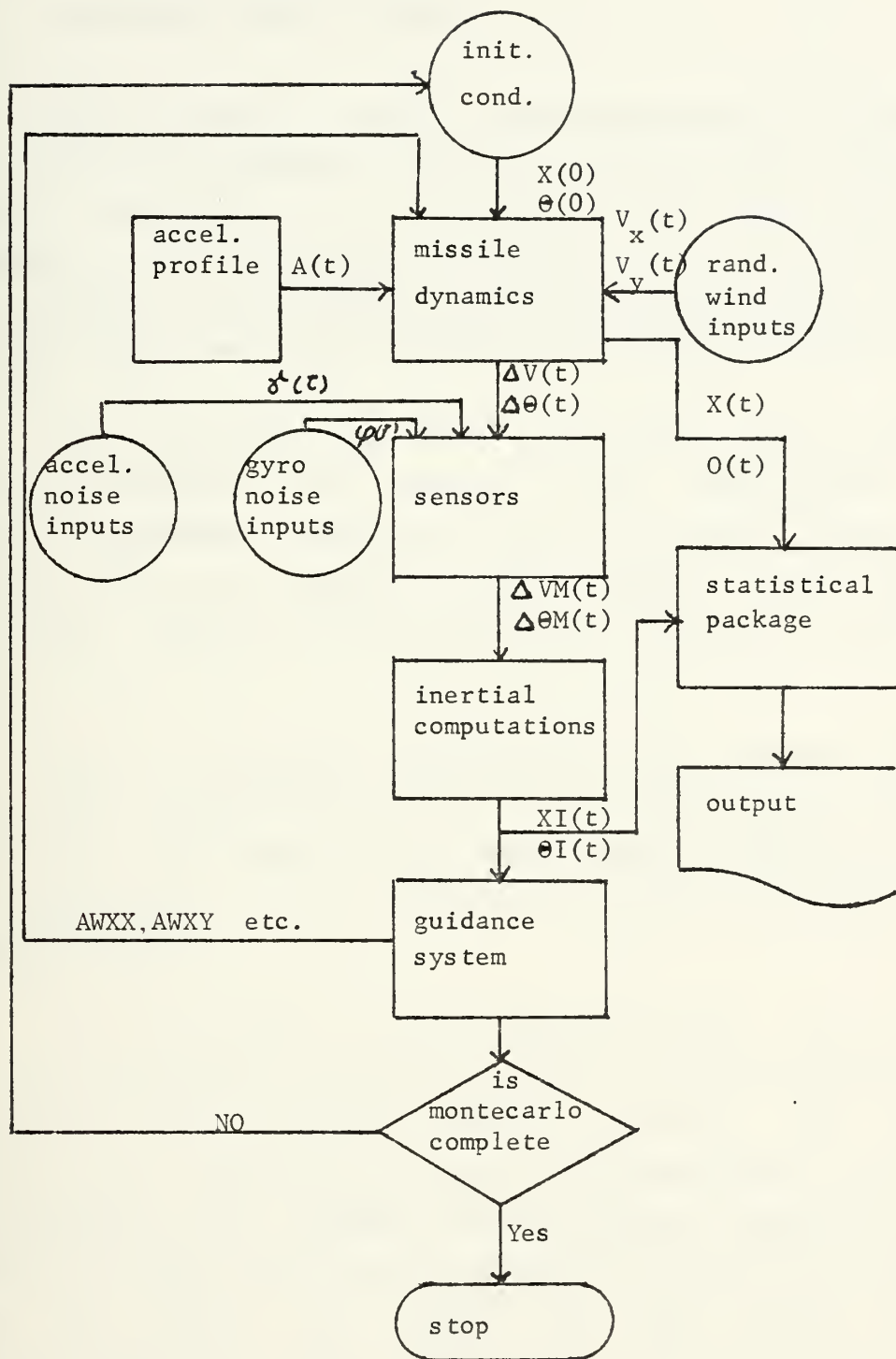


Figure 8 - ALGORITHM FOR MODEL OF MISSILE DYNAMICS WITH PURE INERTIAL COMPUTATIONS

5. Position Update System

The position update system(MICRAD) is designed to fix the missile position at various check points along the flight. Currently it is intended for the missile to navigate to each checkpoint inertially and upon arrival fix its position. The missile would then compute the course to the next check point and proceed to navigate to that point.

For purposes of this simulation and in order to reduce complexity and computer time requirements, the missile estimated position was set equal to its actual position at two discrete time steps. The inertial navigation system was thus reset at time=15 sec and time=80 sec.

The projected accuracy of the MICRAD position measurement system is $\sigma = 50.0$ ft at low altitudes. Thus a noise term was added to the measurement of position within the simulation in an effort to retain agreement with empirical data.

The magnitude of the deviation of the position fix is the fundamental argument in ref.2 for the elimination of the Kalman filter from the inertial system. The rationale behind this assertion is that if two estimates of position are available (i.e. Kalman filter position estimate and a MICRAD estimate) then the weighting placed on each estimate would be heavily in favor of the more accurate estimate, logically the MICRAD fix. The optimal mix of the two estimates would then be

$$X = M X_n + (I-M) X_m \quad (21)$$

where

X_n = navigation estimate = $X + p$

p = navigation estimate error

X_m = measurement estimate = $X + r$

r = measurement error

$M = r / (r + p)$

thus

$$X = (r/(r+p))X_n + (p/(r+p))X_m \quad (22)$$

since logically

$$r \ll p$$

$$X = X_m$$

and the Kalman filter estimates are ignored.

From the above, it can be seen that filtering for position is required only in the intervals between observations and that the optimal mix of filtered position and observed position reduces to the observed position for highly accurate measurements.

6. Statistical Formulations

The primary measures of system performance for inertial navigation systems are mean of estimation error (i.e. the mean of actual position minus inertially calculated position) and variance of estimation error. The mean of estimation error reflects the result of bias within the inertial system and the variance of estimation error is an indication of system reliability. The simulation program adopted in this study evaluated the mean of position and

velocity as well as their variance at each one second interval along the flight path in addition to the estimation error mean and variance. The final position states were also computed and the mean and variance presented separately. It was felt that this information could give a qualitative comparison of the missile performance with and without the Kalman filter installed.

The mean and variance equations for each time interval was computed using standard summation and averaging techniques. Thus

$$\bar{X} = 1/n \sum_{i=1}^N x_i \quad (23)$$

with the inherent assumption that the relative frequency of occurrence is analagous to the probability of occurrence. The variance was computed similarly with

$$S^2 = (1/(n-1)) \sum_{i=1}^N (x_i - \bar{X})^2 \quad (24)$$

IV. KALMAN FILTER

A. INTRODUCTORY REMARKS

Ref.12 discusses various aspects of the philosophy of Kalman filter applications in a very concise manner. Within this discussion the practical limitations of implementation are specified; the foremost limitation being that of a prerequisite knowledge of the exact statistical description for each random signal within the system. This a priori information determines the degree of optimality of the filter.

The filter is said to be optimum in the sense that it generates an unbiased, minimum variance estimate of the states of a linear system from some noisy measurement of a subset of those states. The requirements imposed upon the designer are : exact knowledge of the system dynamics, covariances of initial conditions, and noise inputs. Departure from optimality arises when either estimates of the above quantities are used or approximations to the state equations with lesser state variables comprise the model of system dynamics.

Previous studies indicate that best results are obtained with pessimistic estimations of design parameters and linearization of non-linear systems if possible. The pessimistic estimate of design parameters reduce the sensitivity of the design to deviations within the system while linearization results in off-line gain calculations

which significantly reduce the computational requirements.

The linearization technique analysis of this study indicates that the model of the system error terms are most critical in the estimation of theta. The cross-range and down-range position error being most dependent on these terms. Since digital filtering computational requirements increase roughly as the square of state variables, efficiency dictates that the number of filtered variables be minimized. Therefore it was felt that the best usage of Kalman filter techniques would be in the estimation of theta.

This evaluation is supported by various references (6,8) most notably ref.6.

B. GENERAL THEORY

Given a plant characterized by the linear discrete equations:

$$\begin{aligned}\underline{X}(k+1) &= \underline{\Phi}(k+1,k)\underline{X}(k) + \underline{\Delta}(k+1,k)\underline{U}(k) + \underline{W}(k) \\ \underline{Z}(k) &= \underline{C}(k)\underline{X}(k) + \underline{V}(k)\end{aligned}\tag{25}$$

where

$\underline{X}(k)$	Column matrix of states
$\underline{\Phi}(k+1,k)$	state transition matrix for time k to time k+1
$\underline{\Delta}(k+1,k)$	Forcing transfer function
$\underline{W}(k)$	Process noise term
$\underline{Z}(k)$	Matrix of observations
$\underline{V}(k)$	Measurement noise

With the appropriate assumptions concerning zero mean noise terms and knowledge of the covariance of initial conditions, the optimal estimate of the state vectors at time k can be arrived at through suitable use of a Kalman filter. This estimate will be characterized by a minimum variance of estimation error as its criteria for optimality. The Kalman filter equations are:

$$\begin{aligned}
 \underline{G}(k) &= \underline{P}(k/k-1) \underline{C}(k)^t [\underline{C}(k) \underline{P}(k/k-1) \underline{C}(k) + \underline{R}(k)]^{-1} \\
 \underline{P}(k/k) &= [\underline{I} - \underline{G}(k) \underline{C}(k)] \underline{P}(k/k-1) \\
 \underline{P}(k+1/k) &= \underline{\Phi}(k+1, k) \underline{P}(k/k) \underline{\Phi}^t(k+1, k) + \underline{Q}(k) \\
 \underline{X}(k/k) &= \underline{X}(k/k-1) + \underline{G}(k) [\underline{Z}(k) - \underline{C}(k) \underline{X}(k/k-1)] \\
 \underline{X}(k+1/k) &= \underline{\Phi}(k+1, k) \underline{X}(k/k) + \underline{\Delta}(k+1, k) \underline{U}(k)
 \end{aligned} \tag{26}$$

where the notation $(k+1/k)$ implies the estimate at time $k+1$ given time k .

C. SPECIFIC THEORY

The current method of filtering inertial navigation systems is by building the filter specifically around a given error model of the gyro. The most commonly used error model is a Gauss-Markov drift model described by both correlated noise and bias terms.

$$\dot{D} = - (1/\gamma) D + r \tag{27}$$

where D is the instantaneous value of error, γ is the correlation time and r is a bias term. Statistically the time function has been found to be roughly equivalent to a random walk model such that

$$\dot{D} = w + r \tag{28}$$

where W is a white noise term. Since the above drift is colored noise it has been handled previously by incorporating it in the state matrix such that it is part of the estimation process. The state equations would then become

$$\begin{aligned}\dot{\theta}_1 &= \theta_2 \\ \dot{\theta}_2 &= 0 \\ \dot{D} &= w + r \\ Z &= \theta_2 + \dot{D}\end{aligned}\quad (29)$$

Thus the Kalman filter would be designed for a 3 state vector vice the needed two states. Also the presence of process noise requires a non zero steady state value for the gains of all equations in the update portion of the Kalman filter. This means that improper choice of parameters will bias the estimator for all time

The overriding consideration of this study being the desire to maintain as few state variables as possible prompted a closer look at the above method of analysis.

The inertial navigator had been modeled as a first order system earlier (sect. I)

$$\Theta I(k+1) = \Theta I(k) + \Theta M(k) \quad (30)$$

however the Kalman filter requires an observation matrix and the above model uses the observation matrix $\Delta \Theta M(k)$ as a forcing function therefore for filtering purposes the rate variables had to be redefined and a new state added.

$$\begin{aligned}\Theta I_1(k+1) &= \Theta I_1(k) + \Theta I_2(k) + \Delta \Theta M \\ \Theta I_2(k+1) &= \Theta I_2(k)\end{aligned}\quad (31)$$

The new state variable ΘI_2 is to represent a small

angular rate which should be zero under normal conditions. Notice that the forcing function is solely upon the angular position and not upon the rate variable. Now the observation is the output of the gyro which is

$$\Delta \theta_M(k) = \Delta \theta(k) + \theta I_2(k) * \Delta t + \dot{\varphi}(k) * \Delta t \quad (32)$$

where $\dot{\varphi}$ is the noise term for the gyro given by

$$\dot{\varphi}(k) = E_0 + G(k) \quad (33)$$

E_0 is a constant bias and $g(k)$ is a white noise term. The noise equation then for unit time intervals are

$$\Delta \varphi(k) / \Delta t = E_0 + G(k) = \Delta \varphi(k) \quad (34)$$

and the observation equation is

$$\begin{aligned} \Delta \theta_M(k) &= \Delta \theta(k) + \theta I_2(k) + \Delta \varphi \\ &= \theta(k) + \theta I_2(k) + E_0 + G(k) \end{aligned} \quad (35)$$

Now $\Delta \theta$ is a known forcing function generated by the inertial navigator therefore a new observation can be defined as

$$\begin{aligned} Z^*(k) &= \Delta \theta_M(k) - \Delta \theta(k) \\ &= \theta I_2(k) + E_0 + G(k) \end{aligned} \quad (36)$$

thus the discrete state equations are

$$\begin{aligned} \theta I_1(k+1) &= \theta I_1(k) + \theta I_2(k) + \Delta \theta(k) \\ \theta I_2(k+1) &= \theta I_2(k) \\ Z^*(k) &= \theta I_2(k) + E_0 + G(k) \end{aligned} \quad (37)$$

Now if the constant bias term E_0 is added to the $\theta I_2(k)$ and redefined as

$$\theta I_2'(k) = \theta I_2(k) + E_0 \quad (38)$$

Then it can be seen that the $\Theta I_2(\kappa)$ term takes on the significance of an estimated drift and the state equations then assume the form

$$\begin{aligned}\Theta I_1(k+1) &= \Theta I_1(k) + \Theta I_2'(k) + \Delta \theta(k) \\ \Theta I_2(k+1) &= \Theta I_2'(k) \\ Z^*(k) &= \Theta I_2'(k) + G(k)\end{aligned}\tag{39}$$

Where $g(k)$ is white noise and the state equations are now in standard form for Kalman filtering.

It was necessary to add one more state per gyro simulated but this brings the total state vector to only 12 which is still a net savings in state variables.

A block diagram of the algorithm is given in fig.09.

D. KALMAN FILTER RESULTS

The general Kalman equations of part B above were applied to the angular state variables in an off-line calculation by applying pessimistic estimates of initial condition variance such that

$$\sigma^2_{\theta_1(0)} = 1.00 \times 10^{-03}\tag{40}$$

$$\sigma^2_{\theta_2(0)} = 1.000 \times 10^{-03}$$

and the variance on the measurement noise to be

$$\sigma^2(\varphi) = 1.000 \times 10^{-03}$$

The resulting gains were found to be simple time

functions such that at time $T=K \Delta t$

$$\begin{aligned} G_1(k) &= 1 - (2/k+1) \\ G_2(k) &= 1/k+1 \end{aligned} \quad (41)$$

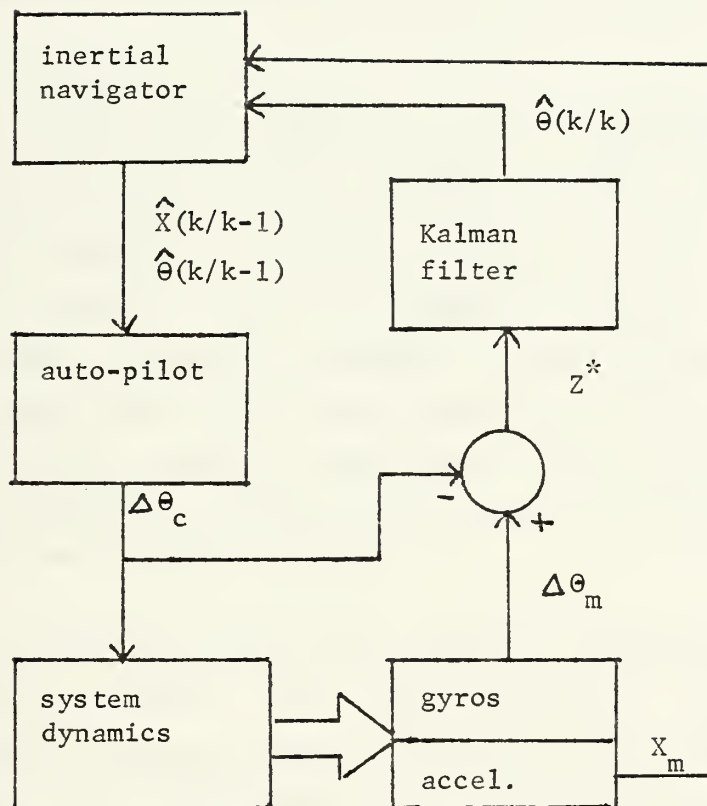


Figure 9 - ALGORITHM FOR PROPOSED KALMAN FILTER IMPLEMENTATION

V. ANALYSIS OF RESULTS

A. VERIFICATION OF RESULTS

Due to the high degree of simplification of plant dynamics, the basic plant model is felt to be weak. The linear dynamics applied to the given g profile produced a trajectory similar to the expected flight profile of the ALVRJ but (due to its simplified nature) with several defects. The descent produced a net increase in forward velocity after level off which would not be the case for a true non-linear model. This defect can be accommodated by the inertial system and hence was not felt to be detrimental to the purposes of this study.

Further work in this area would be recommended in order that basic defects in the missile such as thrust mis-alignment, process noise in control actuators and sensor misalignment be introduced into the simulation. A better tracking scheme could easily be instituted such that the guidance algorithm could institute a more efficient trajectory.

It was felt that for purposes of this study the plant model provided a reasonable approach to ALVRJ simulation. The time of flight and velocity profile are within the same general order of magnitude as the more complex simulations provided by ref.1 and correspond with physical intuition as to proper missile performance.

B. VERIFICATION OF ATIGS SIMULATION

It was felt that the simulation of the ATIGS would be accurate for purposes of this study if the error growth rate and the standard deviation growth rate incurred by the model were close to the observed quantities set forth in ref.1 and ref.3. Reference 3 stated that observed drift in the ATIGS test unit was approximately 1 nm /hr in ground test and 4 nm /hr. in airborne test. The simulation results show a numerical average drift of 3.35 nm /hr. which was felt to be in the range of the actual system. Ref.1 indicates that the simulation reported therein had a cross-range standard deviation growth rate of 1400 ft./min. between the first and second reset positions. The simulation within this study had a cross-range standard deviation growth rate of 1459.4 ft./min.

From the above correlation in performance the study proceeded under the assumption that the simulated model of the ATIGS would provide a reasonable background for analysis of position reset and Kalman filter performance in an actual installation.

C. EFFECT OF POSITION RESET ON SIMULATED PERFORMANCE

The unfiltered position reset feature of this simulation had the expected result of drastically decreasing the variance at mid-course termination. The pure inertial navigator must contend with both initial condition errors and integrated boost phase errors which combine to produce large scale variance at mid-course termination. The initial position reset occurs after boost is complete and thus virtually eliminates the initial condition and boost effect

on position. However, as expected, the velocity and angular errors of boost and initial condition still have effects on the final value of position at mid-course termination. The basic ATIGS navigator without position reset was found to have a radial uncertainty of 1608.6 ft. at mid-course termination (see table 3). The addition of position reset to the basic model was found to reduce this uncertainty to 466.9 ft. However these figures reflect the inherent accuracy of the inertial navigator with the very low error terms in the sensors. If the noise terms in the inertial navigator are allowed to have their standard deviations increased by a factor of 5, thus simulating a very noisy inertial navigator, the basic ATIGS model without position reset demonstrates an uncertainty of 8646.5 ft., and after the addition of position reset, 4128.0 ft. Thus it can be seen that, even with a position reset very close to mid-course termination, large uncertainty of position can accumulate due to the magnitude of the velocity errors which accrue throughout the flight.

D. EFFECT OF ADDITION OF LINEAR SUBOPTIMAL KALMAN FILTER

The Kalman filter proposed in this report had the effect of decreasing the variance of simulation behavior along the flight path for all tested situations (see table 2). If cross-range standard deviations is taken as the criterion for performance quality, it can be seen that the filtered performance is readily superior at all noise levels. The "normal" noise level exhibits a radial uncertainty of 248.03 ft. at mid-course termination and the X5 noise level exhibits a radial uncertainty of 1083.1 ft. The fundamental reason for this increase in accuracy is shown in table 4, where the velocity errors at final update position are compared. The filtered velocity estimates, even at the

higher noise levels, are such that mid-course termination position is within much more reasonable bounds.

The overall effect of the Kalman filter proposed in this study then is to reduce the end point variance of missile position at mid-course termination in a significant manner. The unfiltered updates of position are seen to be valuable when used in conjunction with the Kalman filter but do not insure adequate missile performance if high noise levels are encountered throughout the flight.

RUN TYPE	NOISE	D-RANGE ERROR GROWTH	C-RANGE ERROR GROWTH	1-sigma ERROR GROWTH D-RANGE	1-sigma ERROR GROWTH C-RANGE
ATIGS	NORMAL	2.73 nm/hr	1.449 nm/hr	4016 ft/min	1459.4 ft/min
ATIGS	X5	68.64 nm/hr	107.9 nm/hr	14727 ft/min	10728 ft/min
ATIGS & MICRAD	NORMAL	1.849 nm/hr	1.52 nm/hr	4439.8 ft/min	1995.96 ft/min
ATIGS & MICRAD	X5	82.634 nm/hr	83.6 nm/hr	17986 ft/min	10240 ft/min
ATIGS MICRAD & KALMAN	NORMAL	.439 nm/hr	.87 nm/hr	3736 ft/min	664.59 ft/min
ATIGS MICRAD & KALMAN	X5	21.56 nm/hr	14.373 nm/hr	18836 ft/min	3053 ft/min

TABLE 2-SIMULATION PERFORMANCE WITH TYPE OF
INSTALLATION

RUN TYPE	NOISE	MEAN FINAL POSITION 1-sigma	MEAN FINAL C-RANGE POSITION 1-sigma	MEAN FINAL ERROR 1-sigma	MEAN FINAL C-RANGE ERROR 1-sigma
ATIGS	NORMAL	240880 ft	217.72 ft	6029.4 ft	816.9 ft
		1536.7 ft	475.4 ft	3801.2 ft	2964 ft
ATIGS	X5	241930 ft	413.39 ft	29370 ft	1532.1 ft
		8633.9 ft	466 ft	18514 ft	12891 ft
ATIGS & MICRAD	NORMAL	239,610 ft	74.17 ft	307.4 ft	97.5 ft
		407.7 ft	227.7 ft	246.8 ft	222.4 ft
ATIGS & MICRAD	X5	238180 ft	269.72 ft	2716 ft	293 ft
		3564.7 ft	2081.7 ft	1590.8 ft	1107.5 ft
ATIGS MICRAD & KALMAN	NORMAL	240000 ft	63.0 ft	239.77 ft	34.036 ft
		171.5 ft	194 ft	238.5 ft	68.1 ft
ATIGS MICRAD & KALMAN	X5	240540 ft	61.62 ft	1900.7 ft	59.37 ft
		671.2 ft	1018.9 ft	950.7 ft	518.9 ft

Figure 11 - TABLE 3-FINAL POINT PERFORMANCE WITH TYPE OF

INSTALLATION

type run	noise level	velocity error std. deviation
ATIGS	NORMAL	110.4 ft/sec
ATIGS	X5	562.47 ft/sec
ATIGS & MICRAD	NORMAL	104.65 ft/sec
ATIGS & MICRAD	X5	642 ft/sec
ATIGS, MICRAD & KALMAN	NORMAL	62.8 ft/sec
ATIGS, MICRAD & KALMAN	X5	262.47 ft/sec

Figure 12 - TABLE 4-VELOCITY ERROR OF TYPE OF INSTALLATION
AT FINAL CHECKPOINT

APPENDIX A

RESULTS OF ATIGS SIMULATION WITHOUT FILTERING OR POSITION UPDATE

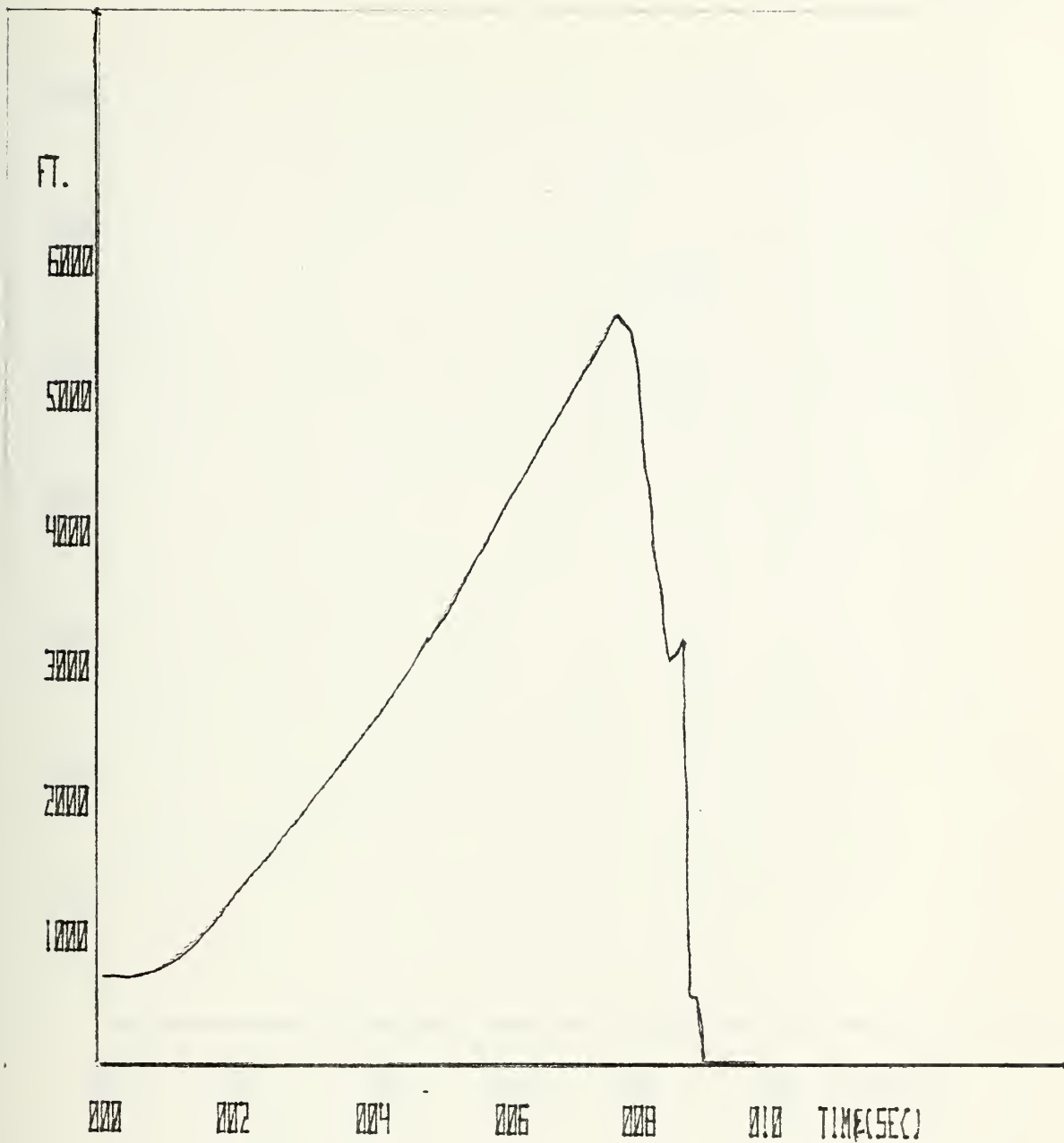


Figure 13 - SQRT OF DOWN RANGE VARIANCE (ATIGS ONLY)

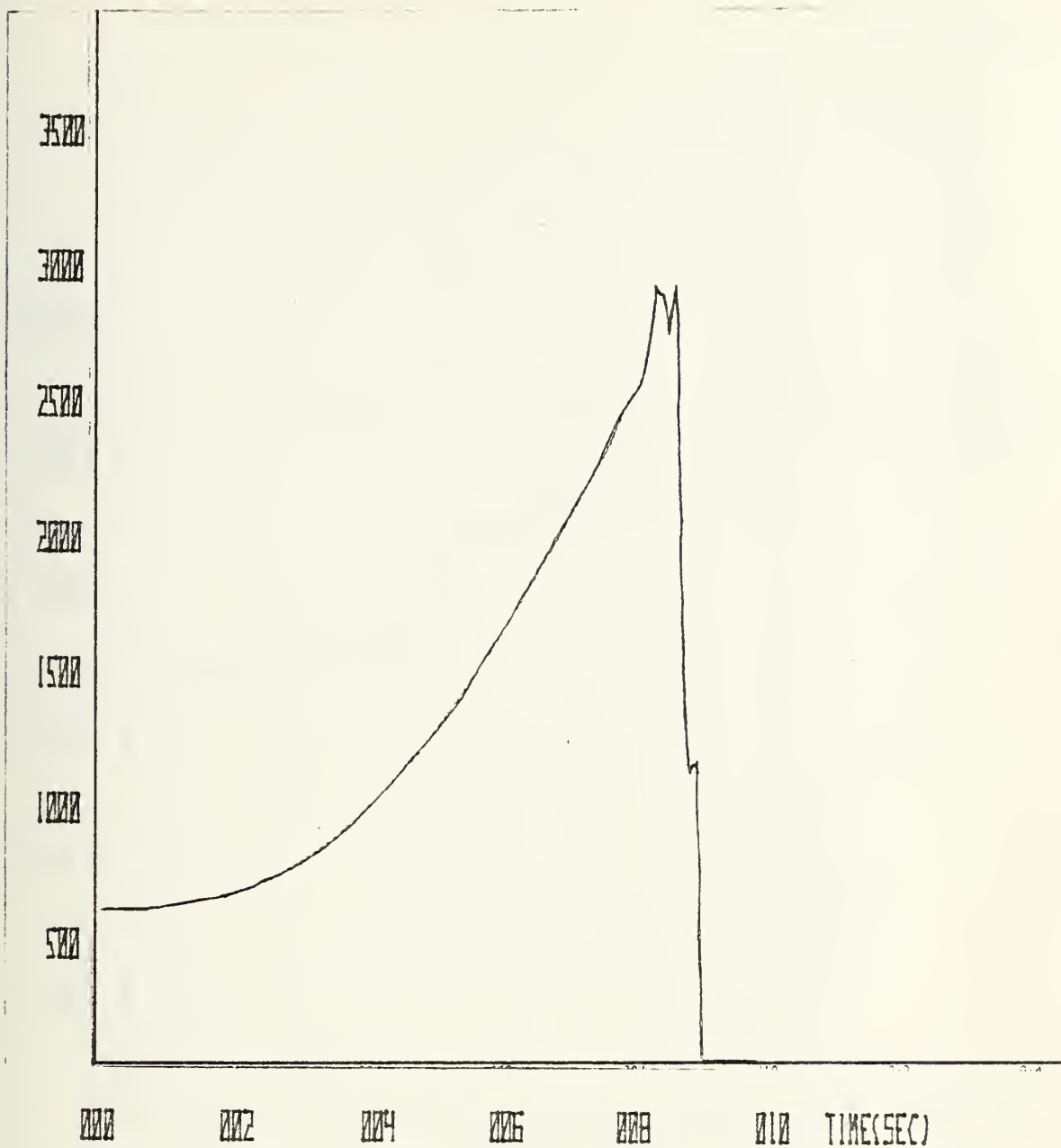


Figure 14 - SQRT OF CROSS RANGE VARIANCE (ATIGS ONLY)

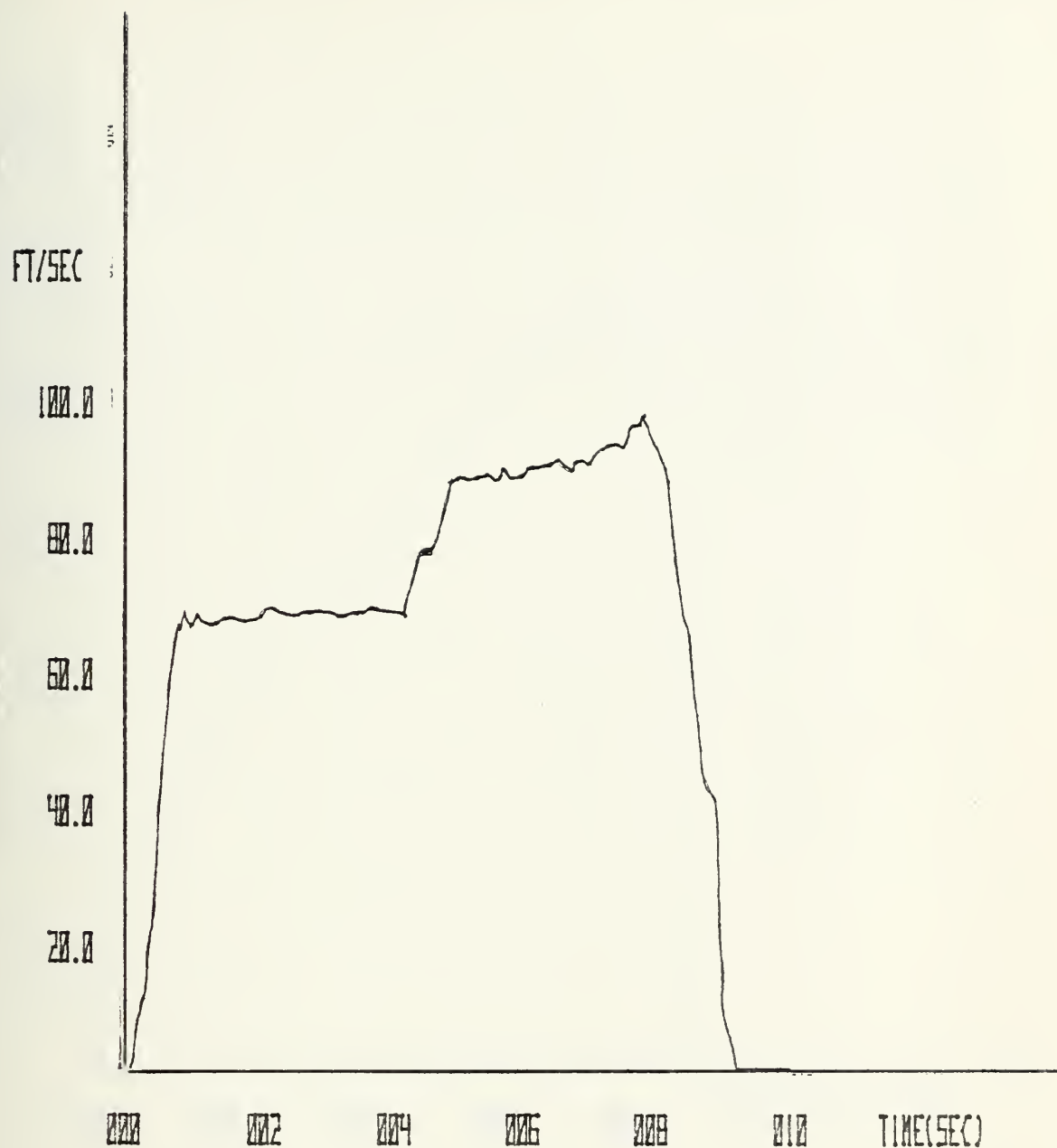


Figure 15 - SQRT• OF DOWN RANGE VELOCITY VARIANCE (ATIGS ONLY)

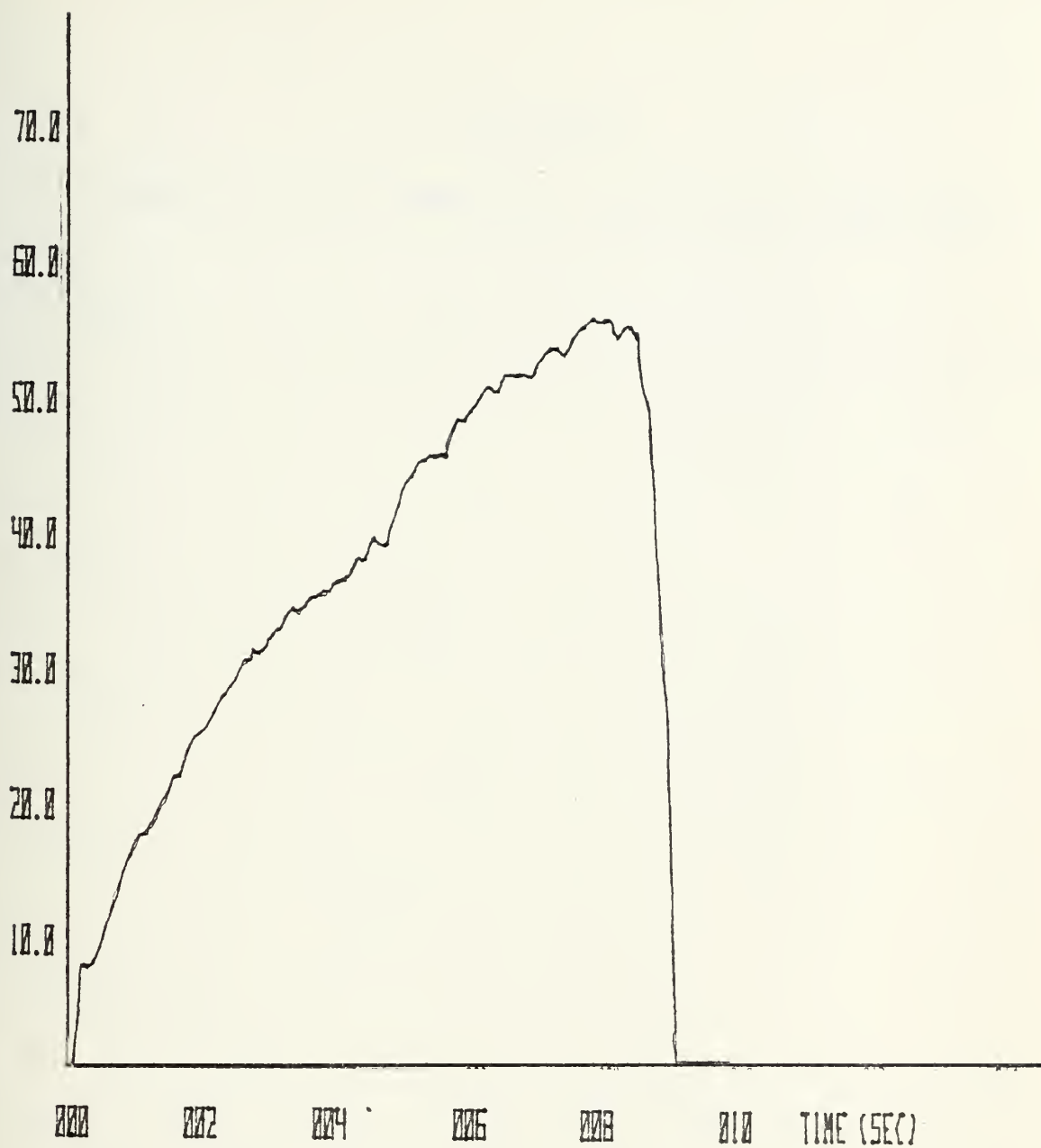


Figure 16 - SQRT OF CROSSRANGE VELOCITY VARIANCE (ATIGS ONLY)

APPENDIX B

RESULT OF ATIGS SIMULATION WITH POSITION RESET ONLY

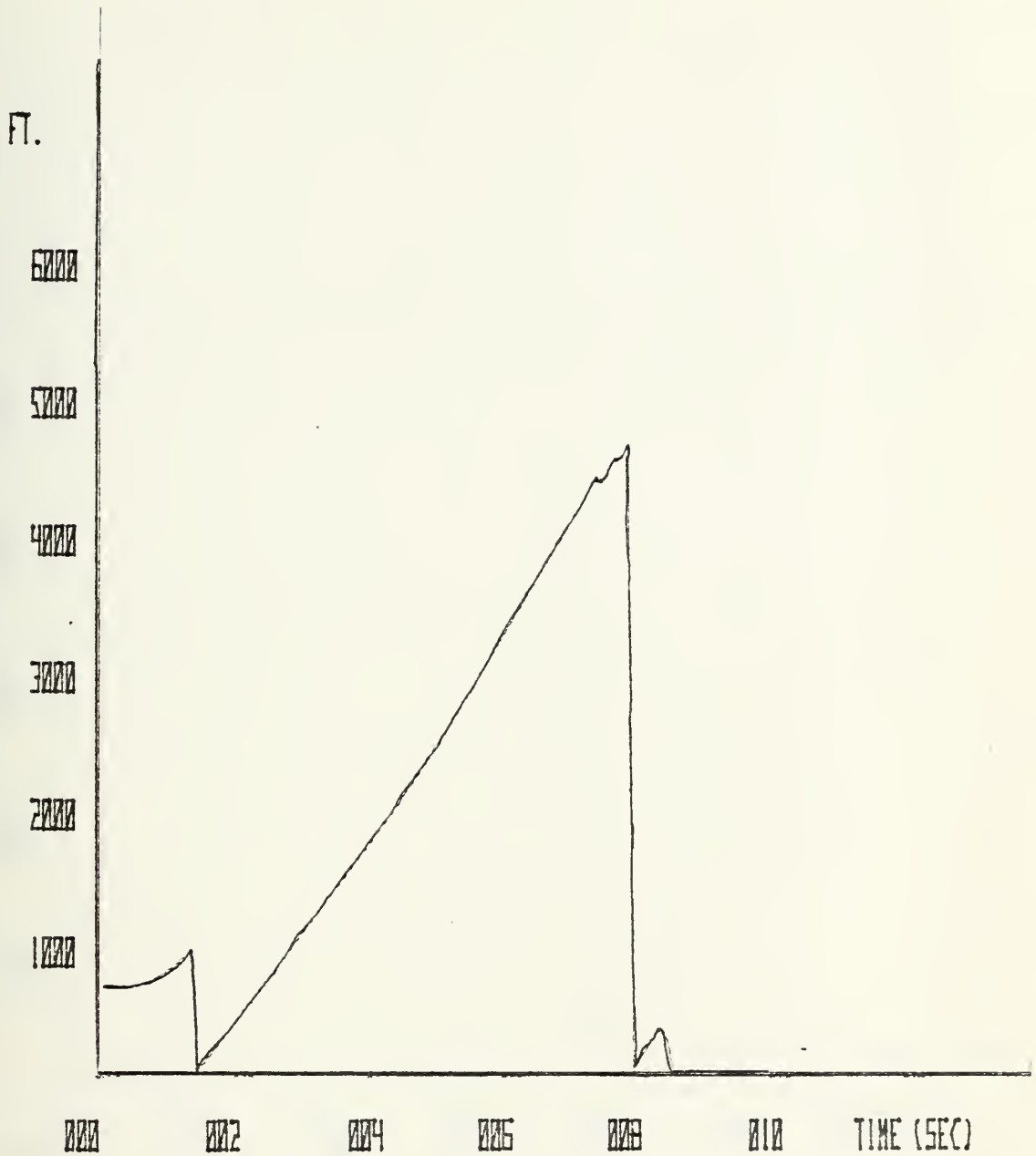


Figure 17 - SQRT OF DOWN RANGE VARIANCE (ATIGS WITH POSIT RESET)

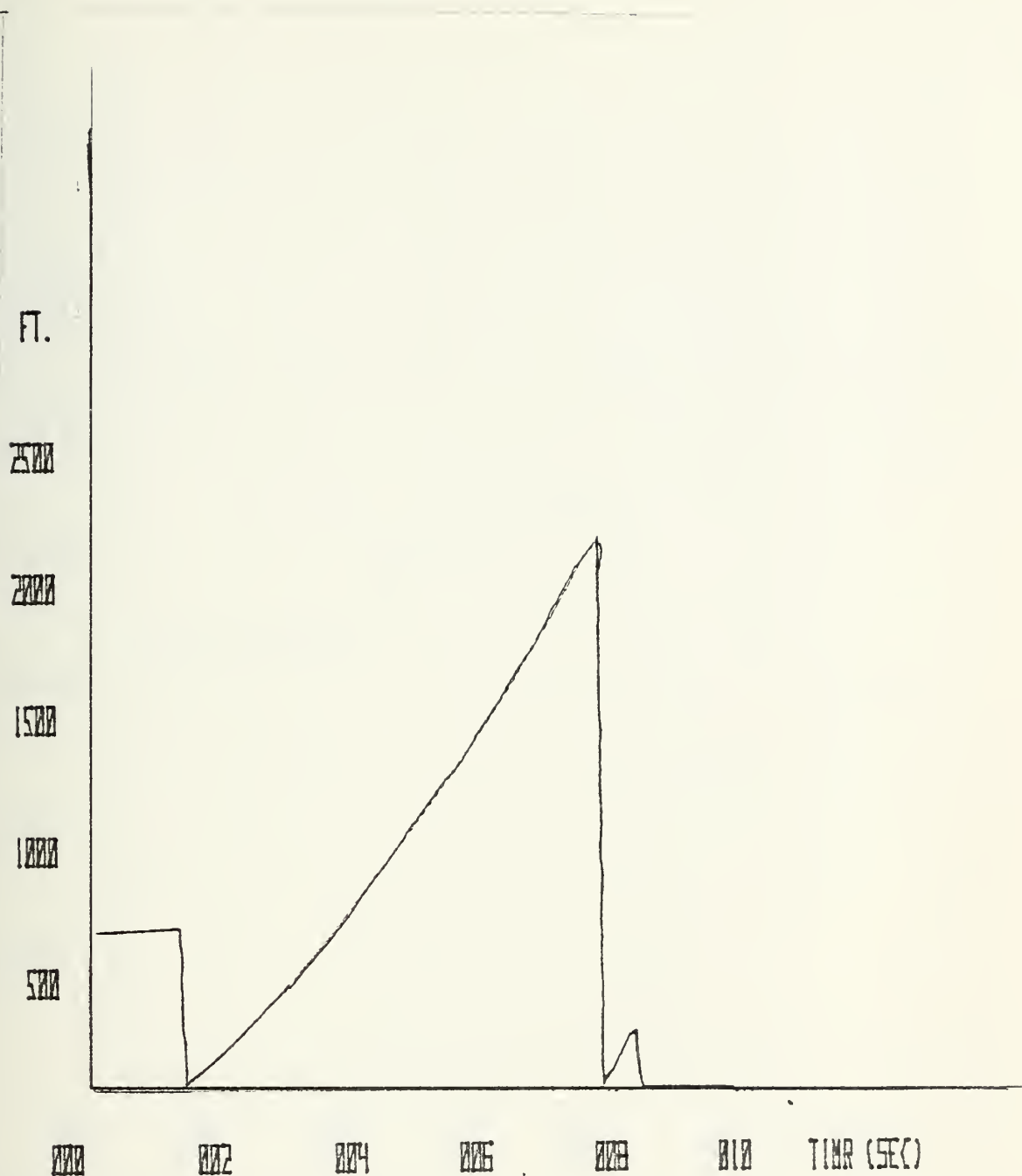


Figure 18 - SQRT OF CROSS RANGE VARIANCE (ATIGS WITH POSIT RESET)

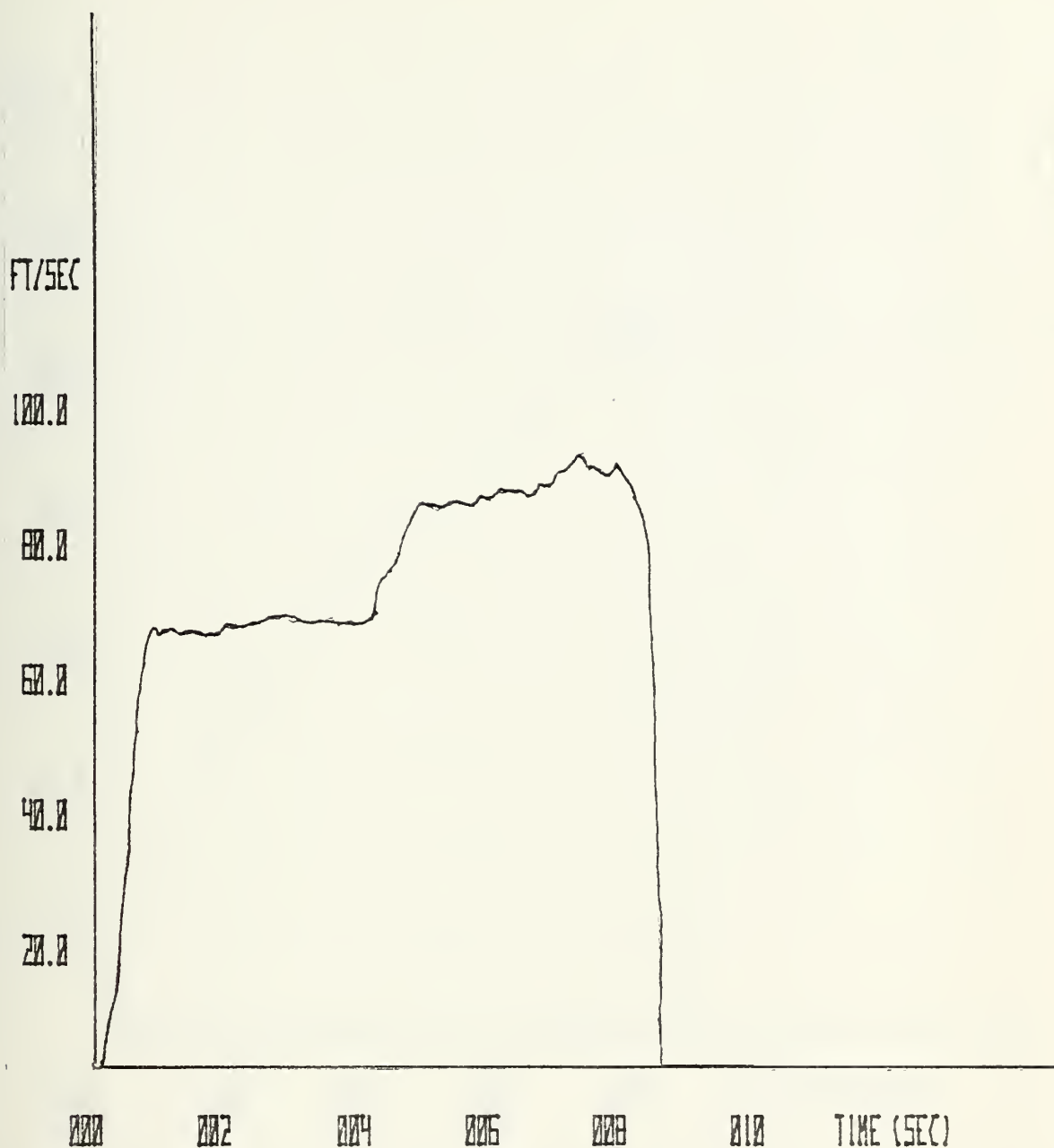


Figure 19 - SQRT OF DOWNRANGE VELOCITY VARIANCE (ATIGS WITH POSIT RESET)

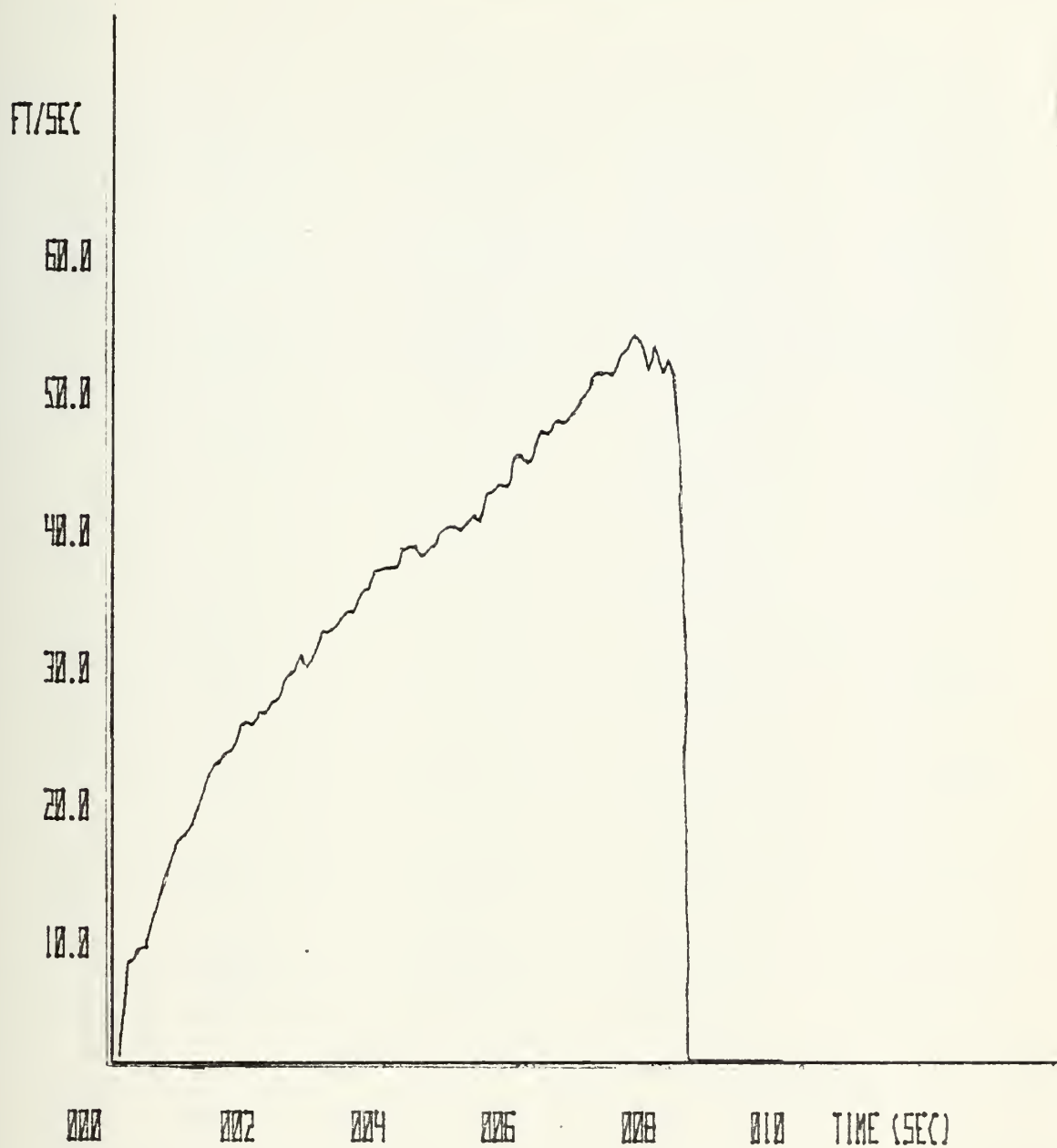


Figure 20 - SQRT OF CROSS RANGE VELOCITY VARIANCE (ATIGS
WITH POSIT RESET)

THE NO. OF GYROSCOPE SIMULATED 3 THE NO. OF ACCEL. SIMULATED 3 SIZE OF THE ENSEMBLE 200
 SIGE0 = 0.330000E-04 SIGW = 0.200000E-02 SIGK = 0.250000E-01 SIGEG = 0.644000E-03 SIGKG = 0.330000E-01 SIGT = 0.582000E-03
 THE UNCERTAINTY IN THE POSITION MEASUREMENT ISO.700000E 01FEET-RADIAL

TIME	MEAN OF TRACK		VAR OF TRACK		MEAN OF ERROR		VAR OF ERROR	
1	X(1)	-0.638470 C2	0.405150 06		-0.638470 02		0.405150 06	
	X(2)	0.700000 C3	0.102220 00		0.300000 02		0.187750 03	
	X(3)	-0.232040 C3	0.364500 06		-0.232040 03		0.364500 06	
	X(4)	0.0	0.0		-0.300000 02		0.187750 03	
	X(5)	0.350000 C5	0.255550 03		0.0		0.0	
	X(6)	0.0	0.0		0.0		0.0	
2	X(1)	0.638270 C3	0.405380 06		-0.640290 02		0.406060 06	
	X(2)	0.764400 C3	0.121900 00		0.297980 02		0.470810 02	
	X(3)	-0.201520 C3	0.364180 06		-0.231610 03		0.364260 06	
	X(4)	-0.172240 C2	0.151320 02		-0.293720 02		0.501740 02	
	X(5)	0.350000 C5	0.255550 03		0.729140 02		0.445240 00	
	X(6)	0.0	0.0		0.145830 01		0.178100 01	
3	X(1)	0.146930 C4	0.405800 06		-0.644710 02		0.408310 06	
	X(2)	0.957600 C3	0.191300 00		0.293560 02		0.120970 03	
	X(3)	-0.172240 C3	0.364080 06		-0.231490 03		0.364550 06	
	X(4)	0.133590 C1	0.242110 01		-0.296840 02		0.573580 02	
	X(5)	0.350000 C5	0.255550 03		-0.407340 01		0.486460 01	
	X(6)	0.0	0.0		-0.110630 00		0.411220 01	
4	X(1)	0.255800 C4	0.405340 06		-0.652690 02		0.410530 06	
	X(2)	0.127940 C4	0.341580 C0		0.287150 02		0.424810 03	
	X(3)	-0.142140 C3	0.364100 06		-0.230790 03		0.365370 06	
	X(4)	0.300580 C1	0.122570 00		-0.291330 02		0.674950 02	
	X(5)	0.350000 C5	0.255550 03		-0.272120 00		0.184670 02	
	X(6)	0.0	0.0		-0.352140 00		0.829830 01	
5	X(1)	0.403350 C4	0.404510 06		-0.664220 02		0.414370 06	
	X(2)	0.173040 C4	0.624660 00		0.279190 02		0.117030 04	
	X(3)	-0.111600 C3	0.364660 06		-0.229170 03		0.367150 06	
	X(4)	0.534360 C1	0.337380 00		-0.286060 02		0.744430 02	
	X(5)	0.350000 C5	0.255550 03		-0.589180 00		0.502010 02	
	X(6)	0.0	0.0		-0.281970 00		0.145920 02	
6	X(1)	0.596000 C4	0.404770 06		-0.684140 02		0.424770 06	
	X(2)	0.218120 C4	0.722520 00		0.268450 02		0.234090 04	
	X(3)	-0.821350 C2	0.365230 06		-0.228580 03		0.369800 06	
	X(4)	-0.256770 C1	0.552200 01		-0.291160 02		0.113980 03	
	X(5)	0.350000 C5	0.255550 03		-0.841830 00		0.111090 03	
	X(6)	0.0	0.0		-0.223340 00		0.229390 02	
7	X(1)	0.827200 C4	0.404430 06		-0.725350 02		0.442920 06	
	X(2)	0.250320 C4	0.130720 01		0.252270 02		0.351820 04	
	X(3)	-0.526020 C2	0.363530 06		-0.228000 03		0.371340 06	
	X(4)	0.105350 C0	0.445850 02		-0.287220 02		0.146570 03	
	X(5)	0.350000 C5	0.255550 03		-0.115790 01		0.219940 03	
	X(6)	0.0	0.0		-0.408870 00		0.393750 02	
8	X(1)	0.108420 C5	0.404020 06		-0.772950 02		0.470640 06	
	X(2)	0.269640 C4	0.151700 01		0.250050 02		0.427450 04	
	X(3)	-0.223300 C2	0.359780 06		-0.226620 03		0.373120 06	
	X(4)	0.163120 C0	0.107410 03		-0.287830 02		0.184000 03	
	X(5)	0.350000 C5	0.255550 03		-0.164620 01		0.415290 03	
	X(6)	0.0	0.0		-0.567720 00		0.683270 02	
9	X(1)	0.135400 C5	0.403300 06		-0.828710 02		0.507980 06	
	X(2)	0.276080 C4	0.159080 01		0.244140 02		0.451430 04	
	X(3)	-0.751750 C1	0.354050 06		-0.225460 03		0.374460 06	
	X(4)	-0.386370 C0	0.151440 03		-0.288320 02		0.242590 03	
	X(5)	0.350000 C5	0.255550 03		-0.211050 01		0.750980 03	
	X(6)	0.0	0.0		-0.360820 00		0.100540 03	

TIME	MEAN OF TRACK			VAR OF TRACK			MEAN OF ERROR			VAR OF ERROR		
10	X(1)	0.162550	C5		0.402730	06	-0.882870	02		0.554220	06	
	X(2)	0.272860	C4		0.155490	01	0.247610	02		0.435590	04	
	X(3)	0.373390	C2		0.347690	06	-0.223770	03		0.375950	06	
	X(4)	-0.570360	C0		0.200250	03	-0.283810	02		0.277950	03	
	X(5)	0.350000	C5		0.255550	03	-0.241200	01		0.128960	04	
	X(6)	0.0			0.0		-0.242200	00		0.130100	03	
11	X(1)	0.189540	C5		0.402380	06	-0.933980	02		0.609750	06	
	X(2)	0.272860	C4		0.155570	01	0.248390	02		0.442800	04	
	X(3)	0.662590	C2		0.341430	06	-0.222880	03		0.378400	06	
	X(4)	-0.599070	C0		0.220790	03	-0.289290	02		0.295690	03	
	X(5)	0.350000	C5		0.255550	03	-0.247700	01		0.210860	04	
	X(6)	0.0			0.0		0.112150	00		0.160780	03	
12	X(1)	0.216520	C5		0.403330	06	-0.987780	02		0.676970	06	
	X(2)	0.272860	C4		0.155640	01	0.246250	02		0.448710	04	
	X(3)	0.623100	C2		0.335270	06	-0.220830	03		0.381300	06	
	X(4)	-0.522420	C0		0.264780	03	-0.279560	02		0.319100	03	
	X(5)	0.350000	C5		0.255550	03	-0.237090	01		0.326170	04	
	X(6)	0.0			0.0		0.100120	00		0.192680	03	
13	X(1)	0.243510	C5		0.402890	06	-0.103470	03		0.750310	06	
	X(2)	0.272860	C4		0.155760	01	0.251480	02		0.434260	04	
	X(3)	0.125560	C3		0.329620	06	-0.219220	03		0.384990	06	
	X(4)	-0.815840	C0		0.288130	03	-0.288120	02		0.370220	03	
	X(5)	0.350000	C5		0.255550	03	-0.218950	01		0.481360	04	
	X(6)	0.0			0.0		0.262660	00		0.227230	03	
14	X(1)	0.270500	C5		0.403460	06	-0.108780	03		0.832670	06	
	X(2)	0.272860	C4		0.155840	01	0.245540	02		0.435970	04	
	X(3)	0.153990	C3		0.325390	06	-0.218880	03		0.390140	06	
	X(4)	-0.151100	C1		0.319170	03	-0.296940	02		0.437770	03	
	X(5)	0.350000	C5		0.255550	03	-0.194500	01		0.679190	04	
	X(6)	0.0			0.0		0.226290	00		0.238460	03	
15	X(1)	0.297480	C5		0.403700	06	-0.105840	00		0.471970	02	
	X(2)	0.272860	C4		0.156200	01	0.243580	02		0.440220	04	
	X(3)	0.181580	C3		0.321080	06	0.104180	01		0.510090	02	
	X(4)	-0.244880	C1		0.367920	03	-0.300890	02		0.488020	03	
	X(5)	0.350000	C5		0.255550	03	-0.168530	01		0.928400	04	
	X(6)	0.0			0.0		0.293150	00		0.293090	03	
16	X(1)	0.324460	C5		0.403750	06	-0.552110	01		0.450450	04	
	X(2)	0.272860	C4		0.156470	01	0.247190	02		0.434630	04	
	X(3)	0.208820	C3		0.317100	06	0.129350	01		0.576390	03	
	X(4)	-0.313670	C1		0.394200	03	-0.294870	02		0.504130	03	
	X(5)	0.350000	C5		0.255550	03	-0.150300	01		0.123860	05	
	X(6)	0.0			0.0		0.715150	-01		0.314050	03	
17	X(1)	0.351460	C5		0.403310	06	-0.997610	01		0.175180	05	
	X(2)	0.272860	C4		0.156640	01	0.252320	02		0.433940	04	
	X(3)	0.236250	C3		0.313830	06	0.205640	01		0.211690	04	
	X(4)	-0.230980	C1		0.431430	03	-0.292760	02		0.531830	03	
	X(5)	0.350000	C5		0.255550	03	-0.101390	01		0.161130	05	
	X(6)	0.0			0.0		0.894640	00		0.348450	03	
18	X(1)	0.378440	C5		0.403640	06	-0.149840	02		0.390460	05	
	X(2)	0.272860	C4		0.156660	01	0.246890	02		0.433990	04	
	X(3)	0.263020	C3		0.310540	06	0.151890	01		0.465990	04	
	X(4)	-0.257280	C1		0.482640	03	-0.302300	02		0.555830	03	
	X(5)	0.350000	C5		0.255550	03	-0.236950	00		0.205310	05	
	X(6)	0.0			0.0		0.671280	00		0.380510	03	
19	X(1)	0.405430	C5		0.404400	06	-0.200050	02		0.697200	05	
	X(2)	0.272860	C4		0.156730	01	0.248580	02		0.444580	04	
	X(3)	0.290350	C3		0.308730	06	0.195350	01		0.835710	04	
	X(4)	-0.302100	C1		0.477730	03	-0.292370	02		0.626260	03	
	X(5)	0.350000	C5		0.255550	03	0.329740	00		0.257160	05	
	X(6)	0.0			0.0		0.462100	00		0.416370	03	

TIME	MEAN OF TRACK			VAR OF TRACK			MEAN CF ERROR			VAR OF ERROR		
20	X(1)	0.432410	C5	0.404990	06		-0.266240	02		0.109490	06	
	X(2)	0.272860	C4	0.156940	01		0.237210	02		0.454210	04	
	X(3)	0.317180	C3	0.307490	06		0.291270	01		0.133830	05	
	X(4)	-0.339630	01	0.517940	03		-0.288450	02		0.647930	03	
	X(5)	0.345260	C5	0.691450	05		-0.199630	01		0.324430	05	
	X(6)	-0.947160	C3	0.275580	06		-0.511410	01		0.201690	04	
21	X(1)	0.459390	05	0.404590	06		-0.336480	02		0.157620	06	
	X(2)	0.272860	C4	0.157170	01		0.238320	02		0.448170	04	
	X(3)	0.343540	C3	0.307290	06		0.387210	01		0.194300	05	
	X(4)	-0.374820	C1	0.559470	03		-0.290930	02		0.643610	03	
	X(5)	0.334340	05	0.275880	06		-0.994400	01		0.429920	05	
	X(6)	-0.123800	C4	0.253040	01		-0.108810	02		0.237540	04	
22	X(1)	0.486380	05	0.404570	06		-0.383450	02		0.214370	06	
	X(2)	0.272860	C4	0.157620	01		0.253310	02		0.452760	04	
	X(3)	0.369650	C3	0.308020	06		0.477450	01		0.267950	05	
	X(4)	-0.419010	C1	0.574420	03		-0.292670	02		0.690800	03	
	X(5)	0.321560	C5	0.276000	06		-0.209020	02		0.579580	05	
	X(6)	-0.123800	C4	0.253040	01		-0.109340	02		0.237390	04	
23	X(1)	0.513370	C5	0.404620	06		-0.425980	02		0.200560	06	
	X(2)	0.272860	C4	0.158020	01		0.251020	02		0.454800	04	
	X(3)	0.394750	C3	0.308780	06		0.406130	01		0.351820	05	
	X(4)	-0.477210	01	0.592420	03		-0.298090	02		0.696040	03	
	X(5)	0.309580	C5	0.276130	06		-0.318580	02		0.775650	05	
	X(6)	-0.123800	C4	0.253040	01		-0.109780	02		0.244690	04	
24	X(1)	0.540360	05	0.404950	06		-0.475990	02		0.355560	06	
	X(2)	0.272860	C4	0.157850	01		0.246090	02		0.456300	04	
	X(3)	0.420160	C3	0.310300	06		0.575020	01		0.447240	05	
	X(4)	-0.538180	01	0.589500	03		-0.292960	02		0.723700	03	
	X(5)	0.297200	C5	0.276260	06		-0.429520	02		0.102160	06	
	X(6)	-0.123800	C4	0.253040	01		-0.112070	02		0.245770	04	
25	X(1)	0.567340	05	0.404960	06		-0.530400	02		0.439830	06	
	X(2)	0.272860	C4	0.159010	01		0.244470	02		0.459270	04	
	X(3)	0.444680	C3	0.313600	06		0.617890	01		0.559280	05	
	X(4)	-0.588500	C1	0.618970	03		-0.301540	02		0.752610	03	
	X(5)	0.284820	C5	0.276390	06		-0.542040	02		0.131760	06	
	X(6)	-0.123800	C4	0.253040	01		-0.112960	02		0.248020	04	
26	X(1)	0.594330	05	0.405040	06		-0.589170	02		0.534060	06	
	X(2)	0.272860	C4	0.158060	01		0.241260	02		0.466140	04	
	X(3)	0.468520	C3	0.317340	06		0.591490	01		0.689630	05	
	X(4)	-0.632440	C1	0.637760	03		-0.306630	02		0.831560	03	
	X(5)	0.272440	C5	0.276530	06		-0.656400	02		0.166250	06	
	X(6)	-0.123800	C4	0.253040	01		-0.115750	02		0.250440	04	
27	X(1)	0.621310	C5	0.405490	06		-0.647900	02		0.638300	06	
	X(2)	0.272860	C4	0.158450	01		0.240990	02		0.468000	04	
	X(3)	0.491580	C3	0.322260	06		0.491480	01		0.837590	05	
	X(4)	-0.714090	01	0.666380	03		-0.309320	02		0.856930	03	
	X(5)	0.260060	C5	0.276670	06		-0.774290	02		0.205970	06	
	X(6)	-0.123800	C4	0.253040	01		-0.120050	02		0.259420	04	
28	X(1)	0.648290	05	0.405530	06		-0.715500	02		0.751410	06	
	X(2)	0.272860	C4	0.158120	01		0.235430	02		0.468380	04	
	X(3)	0.514670	C3	0.328120	06		0.456600	01		0.100520	06	
	X(4)	-0.742300	C1	0.666180	03		-0.305070	02		0.940830	03	
	X(5)	0.247670	C5	0.276820	06		-0.893940	02		0.251090	06	
	X(6)	-0.123800	C4	0.253040	01		-0.119250	02		0.262050	04	
29	X(1)	0.675280	C5	0.405600	06		-0.778970	02		0.874750	06	
	X(2)	0.272860	C4	0.158400	01		0.239490	02		0.474870	04	
	X(3)	0.537210	C3	0.334500	06		0.416820	01		0.118200	06	
	X(4)	-0.806030	C1	0.697710	03		-0.308500	02		0.877460	03	
	X(5)	0.235250	C5	0.276980	06		-0.101380	03		0.301570	06	
	X(6)	-0.123800	C4	0.253040	01		-0.120500	02		0.265420	04	

TIME	MEAN OF TRACK			VAR OF TRACK			MEAN OF ERROR			VAR OF ERROR		
30	X(1)	0.702260	C5	0.406460	06		-0.844740	02		0.100620	07	
	X(2)	0.272860	C4	0.158660	01		0.235340	02		0.462170	04	
	X(3)	0.558520	C3	0.343150	06		0.307230	01		0.137540	06	
	X(4)	-0.875940	01	0.695960	03		-0.307990	02		0.933590	03	
	X(5)	-0.222510	C5	0.277140	06		-0.113620	03		0.357610	06	
	X(6)	-0.123800	C4	0.253040	01		-0.124370	02		0.272820	04	
31	X(1)	0.729240	C5	0.406990	06		-0.920360	02		0.114610	07	
	X(2)	0.272860	C4	0.158660	01		0.228500	02		0.462690	04	
	X(3)	0.579400	C3	0.353510	06		0.178020	01		0.160010	06	
	X(4)	-0.904100	01	0.713830	03		-0.313280	02		0.104190	04	
	X(5)	-0.210530	C5	0.277300	06		-0.126370	03		0.419250	06	
	X(6)	-0.123800	C4	0.253040	01		-0.130600	02		0.278310	04	
32	X(1)	0.756220	C5	0.408420	06		-0.952970	02		0.129530	07	
	X(2)	0.272860	C4	0.159240	01		0.232740	02		0.461250	04	
	X(3)	0.600080	C3	0.365260	06		0.110450	01		0.184250	06	
	X(4)	-0.992770	01	0.746670	03		-0.307540	02		0.103840	04	
	X(5)	-0.198150	C5	0.277470	06		-0.139350	03		0.486550	06	
	X(6)	-0.123800	C4	0.253040	01		-0.128880	02		0.283010	04	
33	X(1)	0.783210	C5	0.409430	06		-0.106070	03		0.145310	07	
	X(2)	0.272860	C4	0.159850	01		0.235430	02		0.458610	04	
	X(3)	0.620070	C3	0.377420	06		-0.240650	01		0.209580	06	
	X(4)	-0.108740	C3	0.754580	03		-0.318320	02		0.106440	04	
	X(5)	-0.185770	C5	0.277650	06		-0.152100	03		0.559690	06	
	X(6)	-0.123800	C4	0.253040	01		-0.126270	02		0.288180	04	
34	X(1)	0.810190	C5	0.409820	06		-0.112550	03		0.161950	07	
	X(2)	0.272860	C4	0.160270	01		0.236070	02		0.459870	04	
	X(3)	0.638100	C3	0.391840	06		-0.185430	01		0.237470	06	
	X(4)	-0.123100	C3	0.760890	03		-0.320800	02		0.110050	04	
	X(5)	-0.173330	C5	0.277830	06		-0.164780	03		0.638340	06	
	X(6)	-0.123800	C4	0.253040	01		-0.127230	02		0.286910	04	
35	X(1)	0.837180	C5	0.409700	06		-0.118800	03		0.179590	07	
	X(2)	0.272860	C4	0.160780	01		0.238260	02		0.463190	04	
	X(3)	0.656030	C3	0.407230	06		-0.420130	01		0.267420	06	
	X(4)	-0.127540	C3	0.777500	03		-0.324730	02		0.113890	04	
	X(5)	-0.161010	C5	0.278010	06		-0.177350	03		0.722610	06	
	X(6)	-0.123800	C4	0.253040	01		-0.124240	02		0.293740	04	
36	X(1)	0.864170	C5	0.410200	06		-0.124400	03		0.198400	07	
	X(2)	0.272860	C4	0.160750	01		0.241710	02		0.471540	04	
	X(3)	0.672700	C3	0.424490	06		-0.658760	01		0.299130	06	
	X(4)	-0.137130	C3	0.804840	03		-0.321110	02		0.114360	04	
	X(5)	-0.148630	C5	0.278200	06		-0.189760	03		0.812720	06	
	X(6)	-0.123800	C4	0.253040	01		-0.123970	02		0.295230	04	
37	X(1)	0.891160	C5	0.410270	06		-0.129510	03		0.217680	07	
	X(2)	0.272860	C4	0.160700	01		0.243940	02		0.452970	04	
	X(3)	0.689480	C3	0.442990	06		-0.824710	01		0.334150	06	
	X(4)	-0.136360	C3	0.801440	03		-0.321150	02		0.123440	04	
	X(5)	-0.136250	C5	0.278400	06		-0.202160	03		0.908760	06	
	X(6)	-0.123800	C4	0.253040	01		-0.123940	02		0.301270	04	
38	X(1)	0.918140	C5	0.410240	06		-0.136250	03		0.237940	07	
	X(2)	0.272860	C4	0.160900	01		0.231390	02		0.460650	04	
	X(3)	0.705560	C3	0.462440	06		-0.108090	02		0.371820	06	
	X(4)	-0.144190	C3	0.790430	03		-0.332120	02		0.126050	04	
	X(5)	-0.123810	C5	0.278600	06		-0.214660	03		0.101160	07	
	X(6)	-0.123800	C4	0.253040	01		-0.126030	02		0.308460	04	
39	X(1)	0.945120	C5	0.410850	06		-0.143100	03		0.259330	07	
	X(2)	0.272860	C4	0.161050	01		0.233180	02		0.463400	04	
	X(3)	0.720410	C3	0.483060	06		-0.146690	02		0.412860	06	
	X(4)	-0.154220	C3	0.809320	03		-0.340590	02		0.135750	04	
	X(5)	-0.111490	C5	0.278900	06		-0.227610	03		0.112090	07	
	X(6)	-0.123800	C4	0.253040	01		-0.133070	02		0.308450	04	

TIME	MEAN OF TRACK			VAR OF TRACK			MEAN OF ERROR			VAR OF ERROR		
40	X(1)	0.972110	05	0.411270	06		-0.150280	03		0.281430	07	
	X(2)	0.272860	04	0.161200	01		0.228580	02		0.458710	04	
	X(3)	0.734950	03	0.505090	06		-0.181160	02		0.456610	06	
	X(4)	-0.159560	02	0.823130	03		-0.332780	02		0.136700	04	
	X(5)	0.991090	04	0.279010	06		-0.241150	03		0.123570	07	
	X(6)	-0.123800	04	0.253040	01		-0.137600	02		0.306650	04	
41	X(1)	0.999090	05	0.411310	06		-0.157790	03		0.304440	07	
	X(2)	0.272860	04	0.161030	01		0.227880	02		0.460100	04	
	X(3)	0.749120	03	0.528130	06		-0.213440	02		0.502470	06	
	X(4)	-0.161820	02	0.817560	03		-0.336730	02		0.137630	04	
	X(5)	0.867280	04	0.279230	06		-0.254900	03		0.135650	07	
	X(6)	-0.123800	04	0.253040	01		-0.137420	02		0.310740	04	
42	X(1)	0.102610	06	0.409400	06		-0.164810	03		0.328240	07	
	X(2)	0.273140	04	0.157330	04		0.227310	02		0.466010	04	
	X(3)	0.762700	03	0.551710	06		-0.247270	02		0.549860	06	
	X(4)	-0.171170	02	0.813800	03		-0.335450	02		0.137260	04	
	X(5)	0.743750	04	0.279390	06		-0.269130	03		0.148190	07	
	X(6)	-0.123150	04	0.761480	04		-0.147290	02		0.306460	04	
43	X(1)	0.105320	06	0.411660	06		-0.173440	03		0.354210	07	
	X(2)	0.274550	04	0.919570	04		0.202900	02		0.539910	04	
	X(3)	0.775400	03	0.578990	06		-0.283500	02		0.602070	06	
	X(4)	-0.171670	02	0.823630	03		-0.333860	02		0.149000	04	
	X(5)	0.622150	04	0.290730	06		-0.285220	03		0.160450	07	
	X(6)	-0.120090	04	0.445860	05		-0.174480	02		0.270940	04	
44	X(1)	0.108070	06	0.445120	06		-0.184620	03		0.381830	07	
	X(2)	0.281570	04	0.413480	05		0.169320	02		0.557840	04	
	X(3)	0.787200	03	0.606490	06		-0.328790	02		0.657450	06	
	X(4)	-0.174350	02	0.811330	03		-0.338750	02		0.148790	04	
	X(5)	0.509800	04	0.360060	06		-0.304190	03		0.172060	07	
	X(6)	-0.104610	04	0.200800	06		-0.204870	02		0.245760	04	
45	X(1)	0.110940	06	0.554890	06		-0.199730	03		0.410870	07	
	X(2)	0.297860	04	0.779560	05		0.130620	02		0.574030	04	
	X(3)	0.798600	03	0.634030	06		-0.377690	02		0.714250	06	
	X(4)	-0.182240	02	0.783220	03		-0.343590	02		0.150120	04	
	X(5)	0.423140	04	0.672890	06		-0.225440	03		0.183320	07	
	X(6)	-0.687030	03	0.378680	06		-0.220170	02		0.242490	04	
46	X(1)	0.113560	06	0.765420	06		-0.214360	03		0.441880	07	
	X(2)	0.313030	04	0.643200	05		0.174440	02		0.614780	04	
	X(3)	0.808950	03	0.660090	06		-0.425480	02		0.772320	06	
	X(4)	-0.198900	02	0.742290	03		-0.341050	02		0.144360	04	
	X(5)	0.371160	04	0.126350	07		-0.344670	03		0.194340	07	
	X(6)	-0.352710	03	0.312440	06		-0.164480	02		0.258650	04	
47	X(1)	0.117120	06	0.967280	06		-0.222760	03		0.474110	07	
	X(2)	0.324550	04	0.232380	05		0.247390	02		0.658410	04	
	X(3)	0.819040	03	0.686680	06		-0.464020	02		0.832440	06	
	X(4)	-0.204050	02	0.695010	03		-0.339860	02		0.147270	04	
	X(5)	0.348580	04	0.177740	07		-0.357350	03		0.204400	07	
	X(6)	-0.988610	02	0.112750	06		-0.890170	01		0.226910	04	
48	X(1)	0.120350	06	0.105720	07		-0.227460	03		0.508620	07	
	X(2)	0.328200	04	0.466510	04		0.272280	02		0.706850	04	
	X(3)	0.821850	03	0.711650	06		-0.505870	02		0.894460	06	
	X(4)	-0.218470	02	0.667740	03		-0.342660	02		0.151910	04	
	X(5)	0.342720	04	0.200320	07		-0.364150	03		0.212920	07	
	X(6)	-0.183760	02	0.226320	05		-0.470460	01		0.181370	04	
49	X(1)	0.123610	06	0.107420	07		-0.229350	03		0.545460	07	
	X(2)	0.329050	04	0.273110	01		0.290070	02		0.734690	04	
	X(3)	0.835890	03	0.737510	06		-0.541870	02		0.961190	06	
	X(4)	-0.226140	02	0.695550	03		-0.334710	02		0.158390	04	
	X(5)	0.341810	04	0.205450	07		-0.368080	03		0.220640	07	
	X(6)	0.198600	00	0.225630	01		-0.315820	01		0.165510	04	

TIME	MEAN OF TRACK			VAR OF TRACK			MEAN OF ERROR			VAR OF ERROR		
50	X(1)	0.126870	C6	0.107470	07		-0.230730	03		0.584260	07	
	X(2)	0.329050	C4	0.274010	01		0.287410	02		0.736220	04	
	X(3)	0.842950	C3	0.762310	06		-0.580630	02		0.103000	07	
	X(4)	-0.229220	C2	0.698580	03		-0.339440	02		0.160890	04	
	X(5)	0.341830	C4	0.205460	07		-0.371140	03		0.228450	07	
	X(6)	0.198600	00	0.225630	01		-0.295030	01		0.168540	04	
51	X(1)	0.130130	C6	0.107520	07		-0.232430	03		0.624340	07	
	X(2)	0.329050	C4	0.274160	01		0.286270	02		0.731840	04	
	X(3)	0.842920	C3	0.788650	06		-0.626420	02		0.110170	07	
	X(4)	-0.237650	C2	0.703480	03		-0.344370	02		0.163060	04	
	X(5)	0.341850	C4	0.205460	07		-0.374280	03		0.236740	07	
	X(6)	0.198600	00	0.225630	01		-0.333410	01		0.182370	04	
52	X(1)	0.133390	C6	0.107630	07		-0.233600	03		0.665430	07	
	X(2)	0.329050	C4	0.280310	01		0.290090	02		0.720860	04	
	X(3)	0.845470	C3	0.811390	06		-0.672270	02		0.117370	07	
	X(4)	-0.250270	C2	0.627810	03		-0.342250	02		0.160600	04	
	X(5)	0.341870	C4	0.205470	07		-0.377260	03		0.245370	07	
	X(6)	0.198600	00	0.225630	01		-0.261760	01		0.181390	04	
53	X(1)	0.136650	C6	0.107530	07		-0.234710	03		0.708300	07	
	X(2)	0.329050	C4	0.280700	01		0.289040	02		0.731900	04	
	X(3)	0.859510	C3	0.831770	06		-0.706090	02		0.124860	07	
	X(4)	-0.257660	C2	0.618040	03		-0.332120	02		0.163690	04	
	X(5)	0.341890	C4	0.205480	07		-0.379550	03		0.254210	07	
	X(6)	0.198600	00	0.225630	01		-0.196220	01		0.182090	04	
54	X(1)	0.139910	C6	0.107600	07		-0.234800	03		0.752710	07	
	X(2)	0.329050	C4	0.280220	01		0.294970	02		0.737560	04	
	X(3)	0.863750	C3	0.852040	06		-0.735310	02		0.132850	07	
	X(4)	-0.261750	C2	0.619390	03		-0.330500	02		0.169130	04	
	X(5)	0.341530	C4	0.204320	07		-0.381580	03		0.263380	07	
	X(6)	-0.726600	C1	0.110630	05		-0.210510	01		0.186200	04	
55	X(1)	0.143170	C6	0.107730	07		-0.235630	03		0.799040	07	
	X(2)	0.329390	C4	0.228150	04		0.207800	02		0.746420	04	
	X(3)	0.867380	C3	0.869720	06		-0.766100	02		0.140820	07	
	X(4)	-0.266380	C2	0.583620	03		-0.331960	02		0.164680	04	
	X(5)	0.341180	C4	0.203710	07		-0.384130	03		0.272860	07	
	X(6)	0.200650	C0	0.226500	01		-0.299580	01		0.193350	04	
56	X(1)	0.146440	C6	0.108310	07		-0.236730	03		0.846330	07	
	X(2)	0.329390	C4	0.228150	04		0.287100	02		0.736820	04	
	X(3)	0.870010	C3	0.889410	06		-0.806840	02		0.149350	07	
	X(4)	-0.273800	C2	0.586210	03		-0.342250	02		0.183120	04	
	X(5)	0.340830	C4	0.204430	07		-0.386960	03		0.282450	07	
	X(6)	-0.726510	C1	0.110840	05		-0.266450	01		0.189660	04	
57	X(1)	0.149700	C6	0.108900	07		-0.237740	03		0.894760	07	
	X(2)	0.325720	C4	0.454230	04		0.285540	02		0.733780	04	
	X(3)	0.871880	C3	0.907950	06		-0.884410	02		0.158360	07	
	X(4)	-0.289840	C2	0.593720	03		-0.333230	02		0.184660	04	
	X(5)	0.339730	C4	0.207220	07		-0.389850	03		0.292350	07	
	X(6)	-0.147350	C2	0.221330	05		-0.311690	01		0.193760	04	
58	X(1)	0.152570	C6	0.109890	07		-0.240040	03		0.945670	07	
	X(2)	0.330410	C4	0.901150	04		0.279020	02		0.754260	04	
	X(3)	0.872250	C3	0.925890	06		-0.884940	02		0.167590	07	
	X(4)	-0.295030	C2	0.562700	03		-0.340610	02		0.188830	04	
	X(5)	0.339000	C4	0.209860	07		-0.393290	03		0.302770	07	
	X(6)	0.199000	00	0.230830	01		-0.376340	01		0.203500	04	
59	X(1)	0.156250	C6	0.112550	07		-0.241820	03		0.998160	07	
	X(2)	0.330410	C4	0.901170	04		0.281340	02		0.755750	04	
	X(3)	0.871530	C3	0.942270	06		-0.921970	02		0.176990	07	
	X(4)	-0.308010	C2	0.561470	03		-0.336150	02		0.186190	04	
	X(5)	0.337500	C4	0.211970	07		-0.396610	03		0.313570	07	
	X(6)	-0.221970	C2	0.329360	05		-0.286860	01		0.208890	04	

TIME	MEAN OF TRACK			VAR OF TRACK			MEAN OF ERRDR			VAR OF ERROR		
60	X(1)	0.159530	C6	0.117010	07		-0.243970	03		0.105210	08	
	X(2)	0.331420	04	0.155110	05		0.279760	02		0.174790	04	
	X(3)	0.870680	03	0.956800	06		-0.955720	02		0.187310	07	
	X(4)	-0.322270	C2	0.548930	03		-0.330610	02		0.204730	04	
	X(5)	0.336050	C4	0.219030	07		-0.399870	03		0.324970	07	
	X(6)	-0.147350	C2	0.221200	05		-0.365110	01		0.225240	04	
61	X(1)	0.162810	06	0.124110	07		-0.245930	03		0.110790	08	
	X(2)	0.332100	C4	0.197440	05		0.285620	02		0.177130	04	
	X(3)	0.867570	C3	0.966370	06		-0.996450	02		0.198080	07	
	X(4)	-0.330980	02	0.565300	03		-0.341800	02		0.207430	04	
	X(5)	0.335330	C4	0.223450	07		-0.403280	03		0.336750	07	
	X(6)	0.197840	C0	0.232730	01		-0.317130	01		0.214940	04	
62	X(1)	0.166110	C6	0.134750	07		-0.246640	03		0.116520	08	
	X(2)	0.332100	C4	0.197440	05		0.289220	02		0.169110	04	
	X(3)	0.864030	03	0.972280	06		-0.103110	03		0.208710	07	
	X(4)	-0.344920	C2	0.561260	03		-0.332720	02		0.201610	04	
	X(5)	0.334600	C4	0.225710	07		-0.406780	03		0.348900	07	
	X(6)	-0.147330	02	0.220520	05		-0.382690	01		0.230750	C4	
63	X(1)	0.169400	C6	0.148120	07		-0.247660	03		0.122420	08	
	X(2)	0.332780	C4	0.238770	05		0.289090	02		0.172650	04	
	X(3)	0.859190	C3	0.976670	06		-0.106240	03		0.219910	07	
	X(4)	-0.356650	02	0.565650	03		-0.334620	02		0.210360	04	
	X(5)	0.333130	C4	0.231410	07		-0.410550	03		0.361450	07	
	X(6)	-0.147340	02	0.221180	05		-0.372800	01		0.227960	04	
64	X(1)	0.172700	C6	0.164080	07		-0.248370	03		0.128410	08	
	X(2)	0.333450	C4	0.279230	05		0.291930	02		0.167570	04	
	X(3)	0.852600	03	0.980070	06		-0.110460	03		0.231950	07	
	X(4)	-0.369120	C2	0.584900	03		-0.343750	02		0.224640	04	
	X(5)	0.331650	C4	0.234490	07		-0.414780	03		0.374270	07	
	X(6)	-0.147400	C2	0.221610	05		-0.473170	01		0.235330	04	
65	X(1)	0.176010	06	0.185110	07		-0.250100	03		0.134590	08	
	X(2)	0.334130	04	0.318890	05		0.282930	02		0.173950	04	
	X(3)	0.845220	C3	0.980480	06		-0.114510	03		0.244410	07	
	X(4)	-0.384110	02	0.577490	03		-0.342940	02		0.222290	04	
	X(5)	0.329800	C4	0.237970	07		-0.419820	03		0.387790	07	
	X(6)	-0.222020	02	0.328840	05		-0.533830	01		0.253180	04	
66	X(1)	0.179320	06	0.212680	07		-0.251430	03		0.140790	08	
	X(2)	0.335140	C4	0.316260	05		0.290220	02		0.162630	04	
	X(3)	0.836160	03	0.979550	06		-0.118380	03		0.257390	07	
	X(4)	-0.405620	02	0.600750	03		-0.343030	02		0.232320	04	
	X(5)	0.327380	C4	0.245100	07		-0.424350	03		0.301820	07	
	X(6)	-0.221980	02	0.329040	05		-0.372210	01		0.242530	04	
67	X(1)	0.182650	C6	0.245780	07		-0.251720	03		0.147220	08	
	X(2)	0.336160	04	0.431670	05		0.295080	02		0.179860	04	
	X(3)	0.824390	C3	0.974380	06		-0.123410	03		0.270700	07	
	X(4)	-0.422050	02	0.654220	03		-0.349680	02		0.231350	04	
	X(5)	0.325740	04	0.251600	07		-0.428860	03		0.416570	07	
	X(6)	-0.147290	C2	0.220550	05		-0.534660	01		0.275660	04	
68	X(1)	0.185950	C6	0.285080	07		-0.251920	03		0.153860	08	
	X(2)	0.336840	C4	0.467430	05		0.298120	02		0.194000	04	
	X(3)	0.811290	C3	0.966630	06		-0.128810	03		0.284480	07	
	X(4)	-0.433110	02	0.682250	03		-0.351680	02		0.232420	04	
	X(5)	0.324640	C4	0.255340	07		-0.433740	03		0.432040	07	
	X(6)	-0.126250	C1	0.110480	05		-0.435750	01		0.264270	04	
69	X(1)	0.189330	06	0.332580	07		-0.252730	03		0.160610	08	
	X(2)	0.337170	C4	0.484790	05		0.288240	02		0.178380	04	
	X(3)	0.797040	03	0.955050	06		-0.133520	03		0.299720	07	
	X(4)	-0.453280	C2	0.714950	03		-0.343810	02		0.237230	04	
	X(5)	0.322720	04	0.262460	07		-0.438250	03		0.448080	07	
	X(6)	-0.312020	02	0.487430	05		-0.467290	01		0.291130	04	

TIME	MEAN OF TRACK			VAR OF TRACK			MEAN CF ERROR			VAR CF ERROR		
70	X(1)	0.192680	C6	0.391240	07		-0.253530	03		0.167620	08	
	X(2)	0.338590	C4	0.617690	05		0.294630	02		0.921980	04	
	X(3)	0.780340	C3	0.938310	06		-0.138760	03		0.313600	07	
	X(4)	-0.469560	C2	0.806200	03		-0.350180	02		0.245630	04	
	X(5)	-0.319570	C4	0.272750	07		-0.442950	03		0.466890	07	
	X(6)	-0.237290	C2	0.378620	05		-0.471660	01		0.283880	04	
71	X(1)	0.196400	C6	0.468160	07		-0.254140	03		0.174850	08	
	X(2)	0.339680	C4	0.730280	05		0.293440	02		0.826160	04	
	X(3)	0.761520	C3	0.917180	06		-0.144360	03		0.329230	07	
	X(4)	-0.494140	C2	0.919310	03		-0.349630	02		0.255960	04	
	X(5)	-0.317220	C4	0.280090	07		-0.448420	03		0.482390	07	
	X(6)	-0.312020	C2	0.487290	05		-0.623880	01		0.308070	04	
72	X(1)	0.199410	C6	0.566810	07		-0.255270	03		0.182240	08	
	X(2)	0.341100	C4	0.855660	05		0.292590	02		0.841650	04	
	X(3)	0.740920	C3	0.091290	06		-0.149210	03		0.345530	07	
	X(4)	-0.512260	C2	0.107170	04		-0.341810	02		0.268990	04	
	X(5)	-0.313810	C4	0.291650	07		-0.455070	03		0.501130	07	
	X(6)	-0.371270	C2	0.543700	05		-0.706070	01		0.330770	04	
73	X(1)	0.202800	C6	0.683080	07		-0.255280	03		0.189840	08	
	X(2)	0.342730	C4	0.926710	05		0.308950	02		0.868180	04	
	X(3)	0.717770	C3	0.857650	06		-0.153020	03		0.362180	07	
	X(4)	-0.549620	C2	0.129420	04		-0.333130	02		0.269720	04	
	X(5)	-0.311210	C4	0.300790	07		-0.460950	03		0.520610	07	
	X(6)	-0.147270	C2	0.220450	05		-0.469020	01		0.311950	04	
74	X(1)	0.206200	C6	0.814350	07		-0.254970	03		0.197680	08	
	X(2)	0.343460	C4	0.953470	05		0.301490	02		0.882550	04	
	X(3)	0.691950	C3	0.815620	06		-0.155650	03		0.379220	07	
	X(4)	-0.575490	C2	0.164960	04		-0.329340	02		0.267580	04	
	X(5)	-0.308470	C4	0.308490	07		-0.464830	03		0.540370	07	
	X(6)	-0.402080	C2	0.643210	05		-0.307800	01		0.312550	04	
75	X(1)	0.209530	C6	0.838960	07		-0.185890	03		0.197120	08	
	X(2)	0.344610	C4	0.105990	06		0.319240	02		0.831290	04	
	X(3)	0.665690	C3	0.769870	06		-0.153350	03		0.398390	07	
	X(4)	-0.608410	C2	0.213680	04		-0.321010	02		0.269800	04	
	X(5)	-0.306490	C4	0.321650	07		-0.462040	03		0.562890	07	
	X(6)	-0.886400	01	0.162350	05		-0.325600	01		0.319320	04	
76	X(1)	0.212550	C6	0.100320	08		-0.183760	03		0.204910	08	
	X(2)	0.345010	C4	0.113570	06		0.327070	02		0.840540	04	
	X(3)	0.633290	C3	0.713380	06		-0.156010	03		0.416450	07	
	X(4)	-0.635260	C2	0.283170	04		-0.327240	02		0.283580	04	
	X(5)	-0.305610	C4	0.327560	07		-0.464870	03		0.584070	07	
	X(6)	-0.886200	01	0.161870	05		-0.239660	01		0.314070	04	
77	X(1)	0.216370	C6	0.119780	08		-0.180550	03		0.212830	08	
	X(2)	0.345400	C4	0.121040	06		0.332350	02		0.830970	04	
	X(3)	0.597330	C3	0.652290	06		-0.158500	03		0.435230	07	
	X(4)	-0.679830	C2	0.380710	04		-0.317740	02		0.289900	04	
	X(5)	-0.304800	C4	0.330800	07		-0.467260	03		0.605890	07	
	X(6)	-0.731760	01	0.111580	05		-0.239460	01		0.320320	04	
78	X(1)	0.219590	C6	0.112870	08		-0.629790	02		0.214270	08	
	X(2)	0.343700	C4	0.967260	05		0.344210	02		0.812840	04	
	X(3)	0.565560	C3	0.522550	06		-0.146470	03		0.449170	07	
	X(4)	-0.718330	C2	0.458840	04		-0.314070	02		0.299460	04	
	X(5)	-0.306590	C4	0.331310	07		-0.417750	03		0.619810	07	
	X(6)	-0.502400	C2	0.811840	05		-0.127070	01		0.335520	04	
79	X(1)	0.222820	C6	0.100850	08		-0.518280	02		0.223480	08	
	X(2)	0.344580	C4	0.106610	06		0.344550	02		0.850580	04	
	X(3)	0.524120	C3	0.527950	06		-0.143470	03		0.468020	07	
	X(4)	-0.783460	C2	0.649940	04		-0.312220	02		0.290840	04	
	X(5)	-0.306320	C4	0.340640	07		-0.369540	03		0.623820	07	
	X(6)	-0.430450	C2	0.712820	05		-0.232210	01		0.390800	04	

TIME	MEAN OF TRACK		VAR OF TRACK		MEAN OF ERROR		VAR OF ERROR	
80	X(1)	0.225910 C6	0.827150 07		0.289120-01		0.614670 02	
	X(2)	0.344140 C4	0.112710 06		0.405620 02		0.826020 04	
	X(3)	0.487960 03	0.463200 06		0.297560 00		0.540870 02	
	X(4)	-0.866640 02	0.633430 04		-0.297380 02		0.269230 04	
	X(5)	-0.307540 C4	0.346340 07		-0.311370 03		0.634900 07	
	X(6)	-0.775660 01	0.114330 05		0.111180 01		0.339500 04	
81	X(1)	0.229210 06	0.871980 07		0.105570 02		0.813860 04	
	X(2)	0.342920 C4	0.956460 05		0.410650 02		0.802960 04	
	X(3)	0.431010 C3	0.402370 06		0.927540 00		0.291620 04	
	X(4)	-0.988650 02	0.102000 05		-0.281770 02		0.291270 04	
	X(5)	-0.311150 04	0.337450 07		-0.263000 03		0.644240 07	
	X(6)	-0.951220 C1	0.174270 05		0.223690 01		0.352610 04	
82	X(1)	0.232040 C6	0.485970 07		0.146710 02		0.300510 05	
	X(2)	0.338560 C4	0.560730 05		0.316290 02		0.759020 04	
	X(3)	0.374100 03	0.322690 06		0.478950 01		0.109900 05	
	X(4)	-0.113660 03	0.135140 05		-0.280520 02		0.269340 04	
	X(5)	-0.326820 C4	0.287080 07		-0.401900 02		0.526670 07	
	X(6)	-0.168430 C2	0.251380 05		0.772270 01		0.288680 04	
83	X(1)	0.234810 C6	0.176510 07		0.124140 02		0.630940 05	
	X(2)	0.334250 C4	0.340230 05		0.352010 02		0.707930 04	
	X(3)	0.302750 C3	0.216080 06		0.763250 01		0.251960 05	
	X(4)	-0.123740 C3	0.201270 05		-0.279570 02		0.280620 04	
	X(5)	-0.346900 C4	0.253820 07		0.279380 03		0.537130 07	
	X(6)	-0.907740 C1	0.137570 05		0.132440 02		0.272000 04	
84	X(1)	0.237850 C6	0.944410 06		0.218920 02		0.100310 06	
	X(2)	0.331240 C4	0.176540 05		0.360850 02		0.644570 04	
	X(3)	0.209250 03	0.105700 06		0.102790 02		0.436540 05	
	X(4)	-0.148000 C3	0.331500 05		-0.282840 02		0.266710 04	
	X(5)	-0.353120 C4	0.218470 07		0.405330 03		0.495660 07	
	X(6)	-0.198010 02	0.294270 05		0.163680 02		0.259420 04	
85	X(1)	0.239610 C6	0.166260 06		0.307370 03		0.609020 05	
	X(2)	0.328050 C4	0.955470 03		0.922180 02		0.239820 04	
	X(3)	0.741750 02	0.513400 05		-0.975580 02		0.494680 05	
	X(4)	-0.168270 C3	0.673910 05		-0.522890 02		0.187150 04	
	X(5)	0.237820 04	0.112580 07		-0.116540 04		0.282860 07	
	X(6)	0.479920 00	0.240020 01		0.113220 02		0.139810 04	

APPENDIX C

RESULTS OF ATIGS SIMULATION WITH POSITION RESET AND KALMAN FILTERING

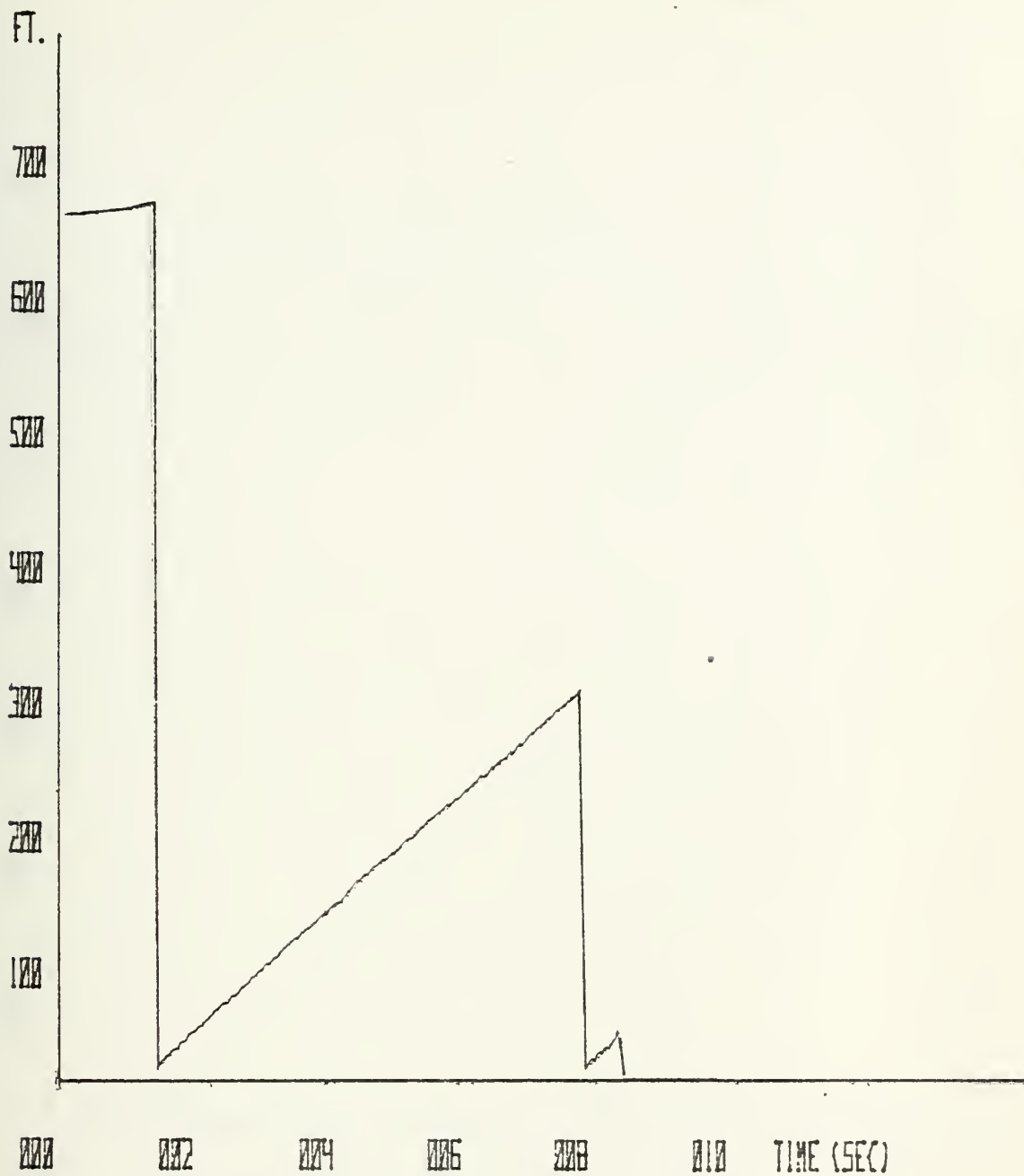


Figure 21 - SQRT OF DOWN RANGE VARIANCE (ATIGS WITH POSITION RESET AND KALMAN FILTERING)

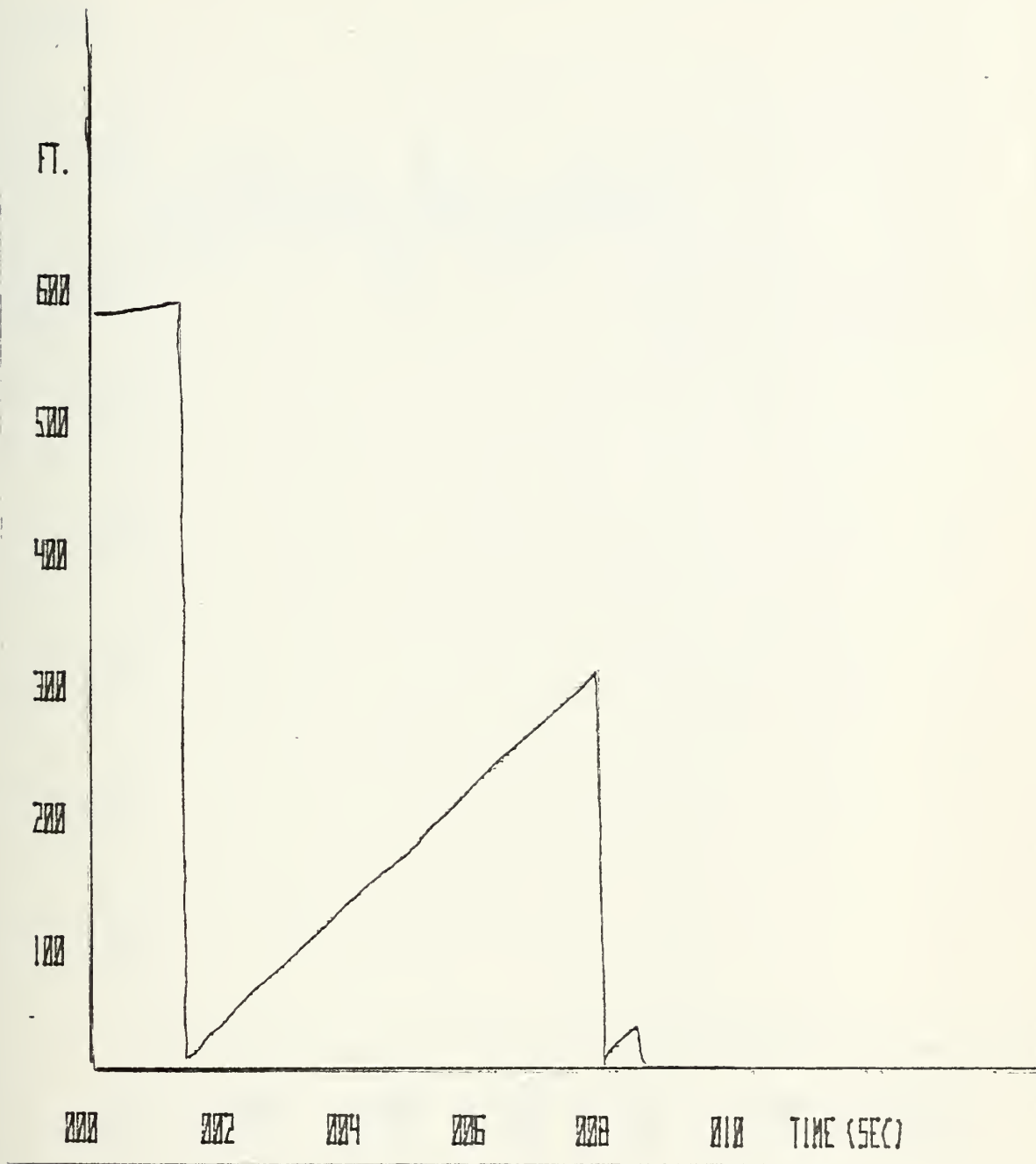


Figure 22 - SQRT OF CROSS RANGE VARIANCE (APIGS WITH POSITION RESET AND KALMAN FILTERING)

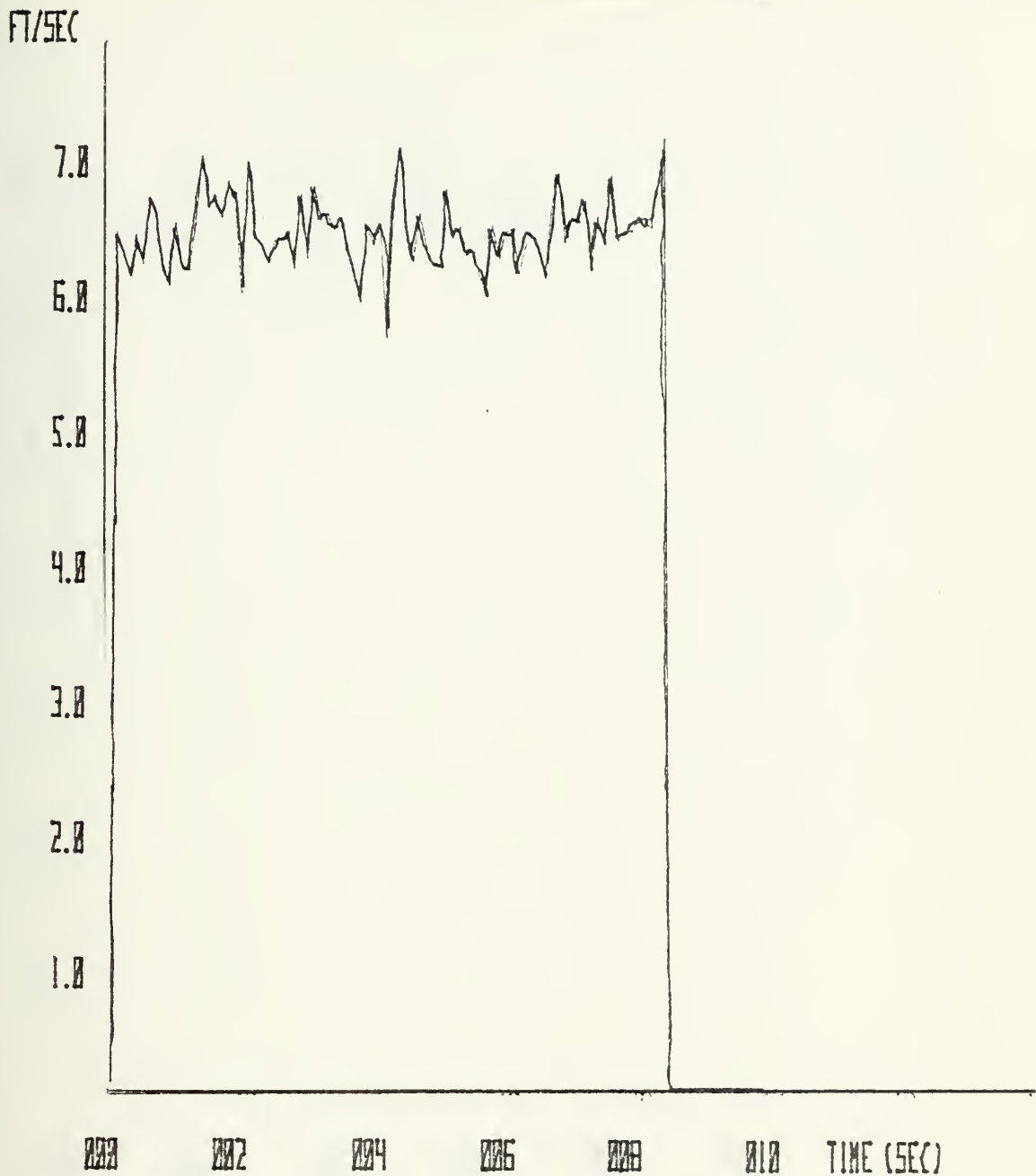


Figure 23 - SQRT• OF DOWN RANGE VELOCITY VARIANCE (ATIGS
WITH POSITION RESET AND KALMAN FILTERING)

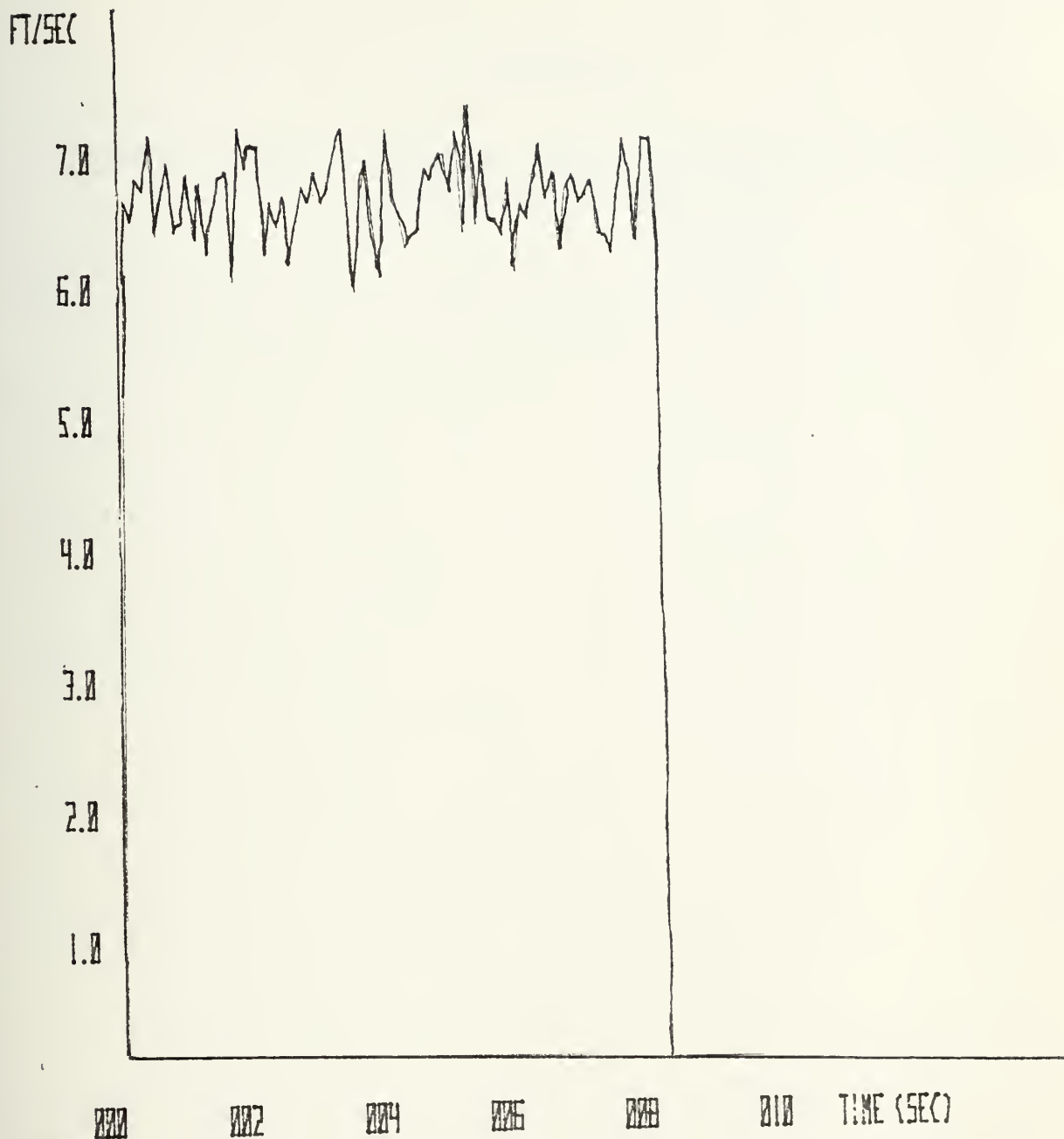


Figure 24 - SQRT• OF CROSS-RANGE VELOCITY VARIANCE (ATIGS
WITH POSTION RESET AND KALMAN FILTERING)

APPENDIX D

RESULTS OF ATIGS SIMULATION WITH POSITION RESET AND KALMAN FILTERING AT X5 NOISE LEVEL

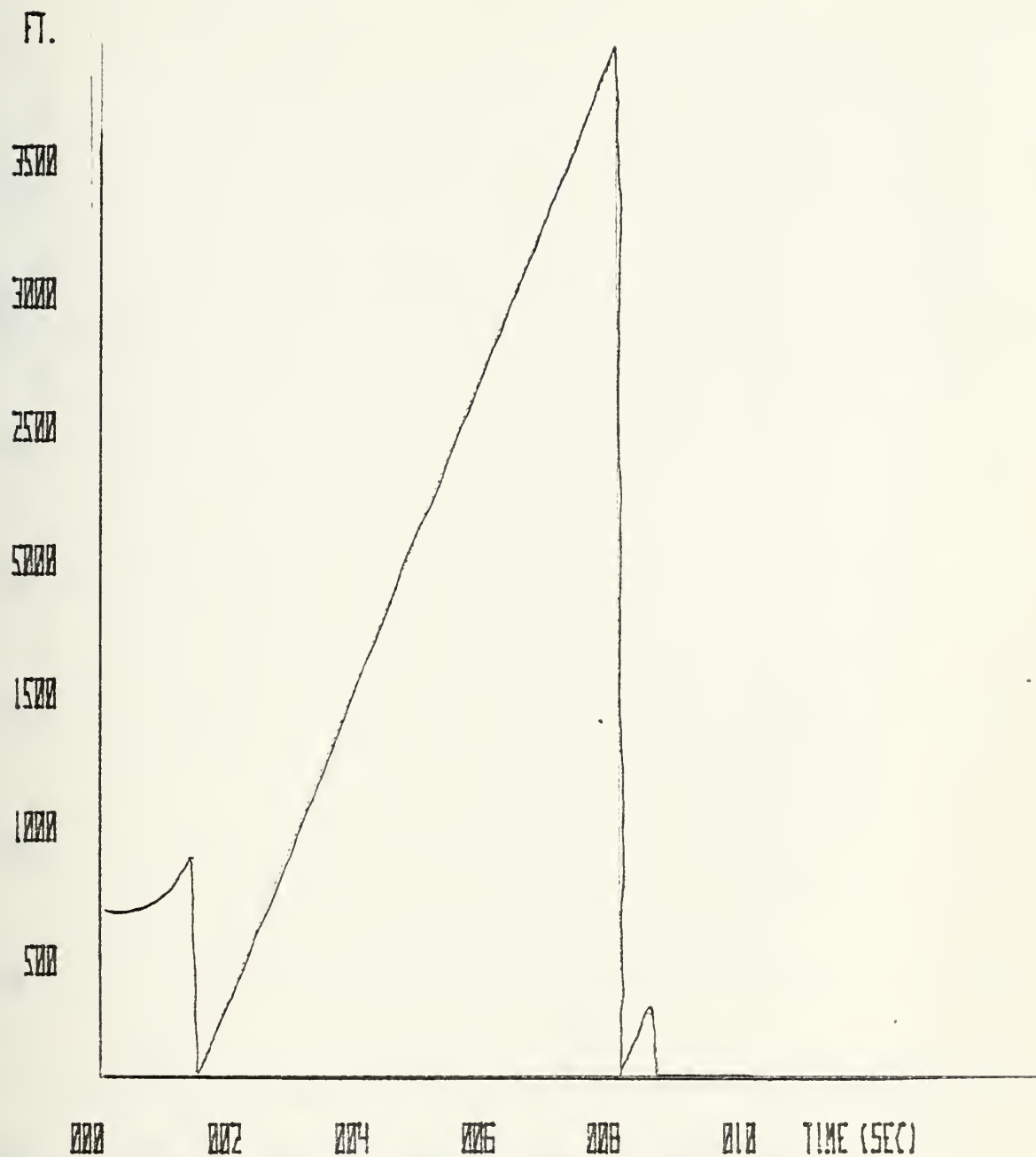


Figure 25 - SQRT• OF DOWN-RANGE VARIANCE (ATIGS WITH POSIT
RESET AND KALMAN FILTERING AT X5 NOISE LEVEL)

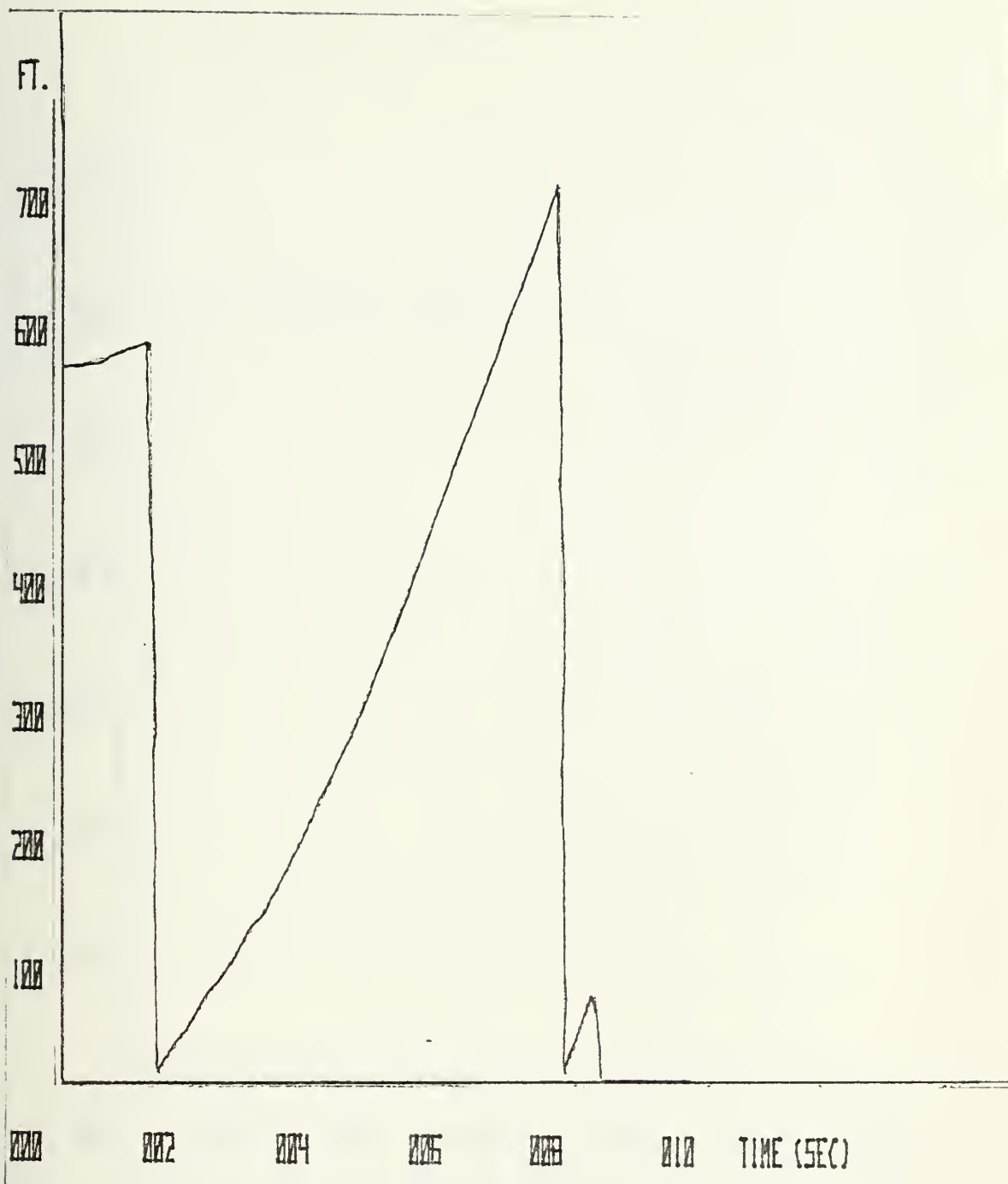


Figure 26 - SQRT• OF CROSS-RANGE VARIANCE (ATIGS WITH POSIT
RESET AND KALMAN FILTERING AT THE X5 NOISE LEVEL)

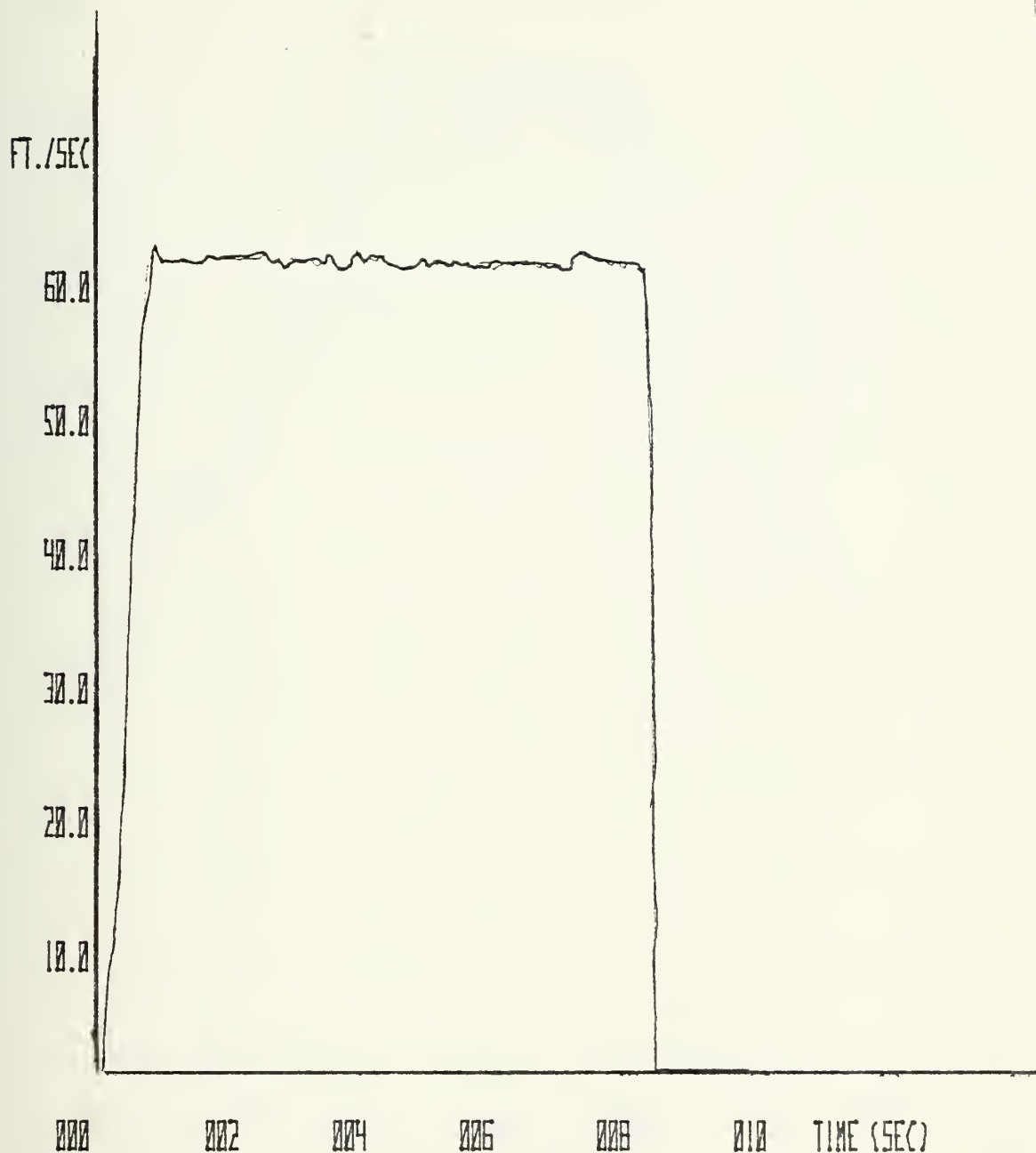


Figure 27 - SQRT. OF DOWN-RANGE VELOCITY VARIANCE (ATIGS WITH POSIT RESET AND KALMAN FILTERING AT THE X5 NOISE LEVEL)

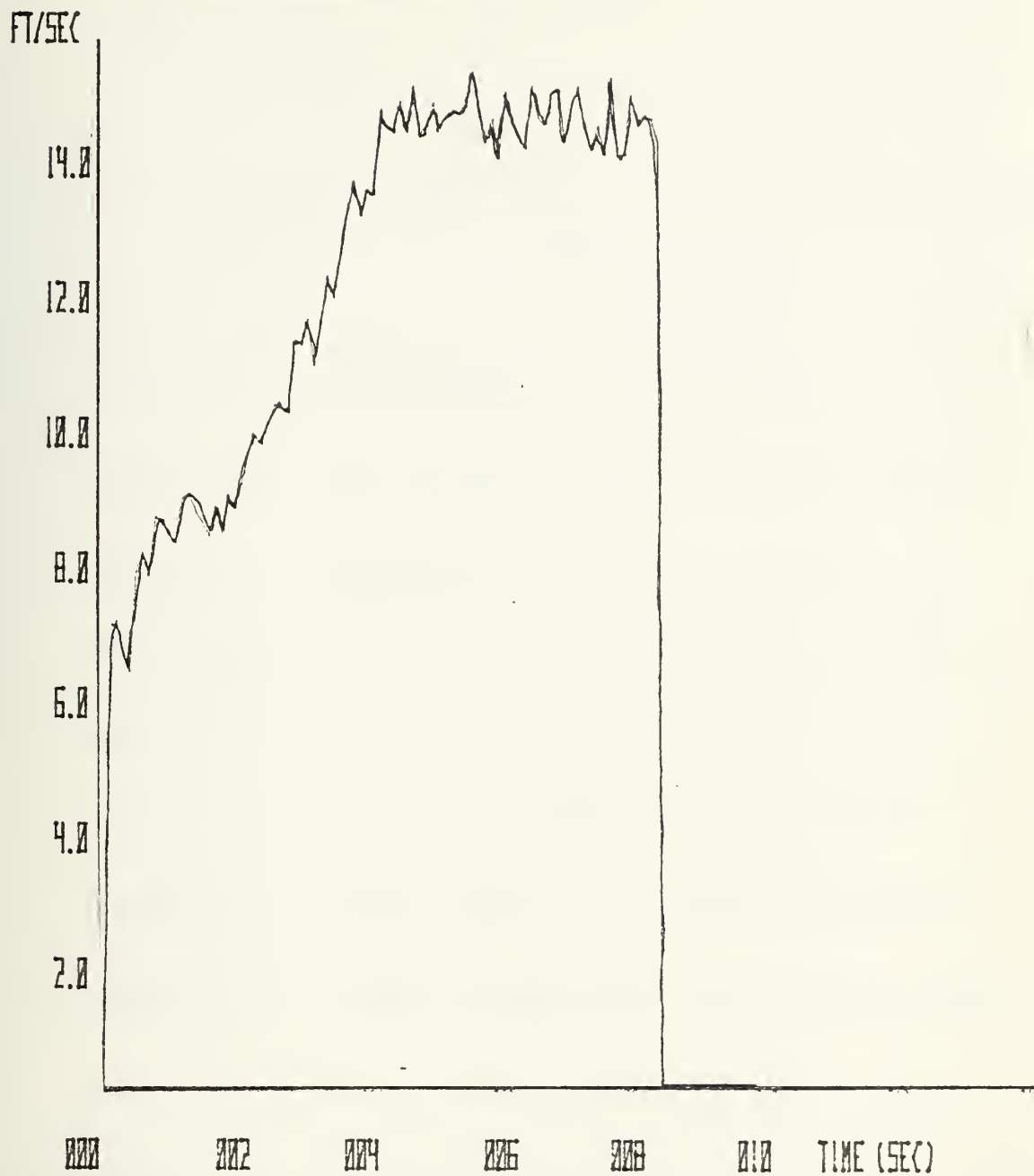


Figure 28 - SQRT• OF CROSS-RANGE VELOCITY VARIANCE (ATIGS WITH POSIT RESET AND KALMAN FILTERING AT THE X5 NOISE LEVEL)

APPENDIX E

PARTIAL LISTING OF SYMBOLS AND NOMENCLATURE OF SIMULATION PROGRAM

X(i,j)	i-th missile state at time j
THETA(i,j)	i-th angular state variable at time j
XI(i,j)	estimated i-th state at time j
THETA(i,j)	estimated i-th angular state at time j
XBAR(i,j)	mean of i-th state at time j
XBVAR(i,j)	variance of i-th state at time j
XMEAN(i,j)	error mean of i-th estimated state
XVAR(i,j)	error variance of i-th estimated state
A(j)	thrust acceleration at time j
AWXY	
AWYY	
AWYZ	
AWZZ	
AWXX	velocity changes due to angular rotation
DELVX	changes in wind components
DELVY	

BETA1	body referenced accelerations
BETA2	
BETA3	
YIX	inertial referenced accelerations
YIY	
YIZ	
XPOS	micrad sensed positions
YPOS	
G1	Kalman gains for filter
G2	
OMEGAX	extra states for Kalman filter
OMEGAY	
XINT	dummy variable for output
PSI	change in drift angle
XFIN(i)	final value of i-th state per track
NTERM	maximum time steps allowed
VXO	velocity of wind down-range
VYO	velocity of wind cross-range
IX	seed number for random number generators
DTHTAX	change in thetax
dthtxm	measured change in thetax
ZNI(j)	total tracks through time j

XIFIN(i)	final value of estimated i-th state per track
XBFIN(i)	mean of XFIN(i)
XIBFN(i)	mean of XIFIN(i)
XBFV(i)	variance of XFIN(i)
N	number of gyros simulated
NNA	number of accelerometers simulated
IENSB	size of ensemble
SIGEO	std.deviation of gyro bias
SIGW	std.deviation of gyro random walk
SIGK	std.deviation of gyro scale factor
SIGEG	std.deviation of accelerometer bias
SIGKG	std.deviation of accelerometer scale factor
SIGT	std.deviation of initial condition on theta

APPENDIX F

SIMULATION PROGRAM


```
// EXEC FORTCLGP, REGION=180K
// FORT.SYSIN DD *
```

```
C      X (1,J)      THE ACTUAL POSITION ALONG THE DOWNRANGE
C      X (2,J)      THE AIRSPEED IN THE DOWNRANGE DIRECTION
C      X (3,J)      NOTE IT IS NOT THE ACTUAL VELOCITY
C      X (4,J)      CROSSRANGE POSITION
C      XI (K,J)     CROSS RANGE AIRSPEED
C      XBAR         THE INERTIALLY COMPUTED STATES ALONG THE
C      XMEAN        DOWN-RANGE/CROSS-RANGE AXIS
C                  THE MATRIX OF MEAN VALUES OF THE MONTE-
C                  CARLO GENERATED TRACKS OF THE X MATRIX
C                  THE MATRIX OF MONTECARLO GENERATED
C                  MEANS OF THE XI MATRIX
```

```
DIMENSION NA (3), EOG (3), WG (3), PSI (150), GAM (3), ZNI (150),
DIMENSION THETA (3, 150), THETA1 (3, 150), G (3), EO (3), ZK (3),
DIMENSION A (150), XFIN (6), XIFIN (6), YF (150), XINT (150), XP
REAL*8 XBFIN (6), XIBFN (6), XBFV (6), XIBFV (6), XM1, XM3
REAL*8 XBAR (6, 155), XBVAR (6, 155), X (6, 155), XI (6, 155)
REAL*8 XMEAN (6, 150), XVAR (6, 150)
DIMENSION ERR (6)
```

```
THIS SECTION READS IN THE VARIOUS NECESSARY SPECIFICAT
"N " THE NUMBER OF GYROS INVOLVED PER SIMULATION
"NNA" THE NUMBER OF ACCELEROMETERS PER SIMULATION
"IENSB" THE NUMBER OF THE ENSEMBLE
```

```
READ (5, 300) N, NNA, IENSB
"SIGEO" IS THE DIVIATION OF THE GYRO BIAS
"SIGW" IS THE DEVIATION OF THE RANDOM WALK CONSTANT
FOR THE GYROS
"SIGK" IS THE DEVIATION OF THE SCALE FACTOR (GYRO)
"SIGEG" IS THE DEVIATION OF THE ACCEL. BIAS
"SIGKG" IS THE DEVIATION OF THE ACCEL SCALE FACTOR
"SIGT" IS THE DEVIATION OF THE INITIAL CONDITION
ON THETA
```

```
READ (5, 310) SIGEO, SIGW, SIGK, SIGEG, SIGKG, SIGT
```

```
"SIGMIC" IS THE DEVIATION OF THE POSITION MEASUREMEN
```

```
READ (5, 310) SIGMIC
WRITE (6, 420) N, NNA, IENSB
WRITE (6, 440) SIGEO, SIGW, SIGK, SIGEG, SIGKG, SIGT
WRITE (6, 500) SIGMIC
CALL OVFLOW
```

```
INDX1=1
INDX2=1
INDX3=1
NDEBG=1
YP=0.0
NTERM=100
IA=1
IB=3
IX=11111
VXO=30.0
VYO=30.0
DO 02 J=1, NTERM
DO 01 K=1, 6
XMEAN (K, J)=0.0
XVAR (K, J)=0.0
XBAR (K, J)=0.0
XBVAR (K, J)=0.0
```

```
01 CONTINUE
02 CONTINUE
```

```
GENERATE THE THRUST ACCELERATION PROFILE
```

```
A (1)=2.0
```



```

A(2)=6.0
A(3)=10.0
A(4)=14.0
A(5)=14.0
A(6)=10.0
A(7)=6.0
A(8)=2.0
A(9)=-1.0
DO 03 I=10,150
03 A(I)=0.0
DO 04 J=1,6
XBFIN(J)=0.0
XIBFN(J)=0.0
XBFV(J)=0.0
04 XIBFV(J)=0.0
DO 05 I=1,NTERM
ZNI(I)=0.0
05 A(I)=A(I)*32.2

```

```

C
C
C      START THE MONTECARLO SIMULATION

```

```

DO 200 NI=1,IENSB
TIMEX=1.0
OMEGAY=0.0
OMEGAX=0.0

```

```

C
C
C      THE PURPOSE OF THIS SECTION IS TO COMPUTE THE INITIAL
      CONDITION FOR THE ACTUAL TRACK OF THE MISSILE.

```

```

CALL RANDU(IX,IY,XFL)
IX=IY
X(1,1)=XFL*2240.0-1120.0
10 CALL RANDU(IX,IY,YFL)
IX=IY
IF((XFL**2+YFL**2).GT.1.0)GO TO 10
X(2,1)=700.0
X(3,1)=YFL*2240.0-1120.0
X(4,1)=0.0
X(5,1)=35000.
X(6,1)=0.0

```

```

C
C      GENERATE THE INITIAL CONDITIONS ON THETA

```

```

CALL SNORM(IX,T,N)
DO 11 J=1,N
THETA(J,1)=0.0
11 THETA(J,1)=T(J)*SIGT

```

```

C
C
C      THE INITIAL SETTING OF THE INERTIAL NAVIGATOR IS
      ZERO POSITION IN DOWN-RANGE AND CROSS-RANGE AND THE
      ACTUAL VELOCITY AND ALTITUDE

```

```

XI(1,1)=0.0
XI(2,1)=670.0
XI(3,1)=0.0
XI(4,1)=30.0
XI(5,1)=35000.0
XI(6,1)=0.0
DIHTAX=0.0
DIHTAY=0.0
DIHTAZ=0.0

```

```

C
C
C
C      THE PURPOSE OF THIS SECTION IS TO PRODUCE THE ACTUAL
      TRACK OF THE SIMULATED MISSILE FOR COMPARISON WITH
      OTHER ESTIMATES OF POSITION
      WIND EFFECTS ARE COMPUTED FIRST

```

```

C
C      THE INITIAL VALUE OF C IS ALWAYS ZERO
C=0.0

```

```

C
C
C      THE VARIOUS RANDOM INPUTS FOR MEASUREMENT DEVICES ARE
      GENERATED

```



```

C      GENERATE GYRO BIAS "EO"
      CALL SNORM (IX,EO,N)
      DO 18 KN=1,N
18      EC (KN) = SIGEO*EO (KN)

C
C
C      GENERATE THE RANDOM SCALE FACTOR "ZK"
      CALL SNORM (IX,ZK,N)
      DO 19 KN=1,N
19      ZK (KN) = SIGK*ZK (KN)

C
C
C      GENERATE RANDOM INPUTS TO ACCELEROMETER PACKAGE

      GENERATE ACCELEROMETER BIAS "EOG"
      CALL SNORM (IX,EOG,NNA)
      DO 20 KN=1,NNA
20      EOG (KN) = SIGEG*EOG (KN)

C
C
      GENERATE ACCELEROMETER SCALE FACTOR "KG"
      CALL SNORM (IX,KG,NNA)
      DO 30 KN=1,NNA
30      WG (KN) = SIGKG*WG (KN)
      DO 100 J=1,NTERM
      ZNI (J) = ZNI (J) + 1.0
      JF1=J+1
      CALL RANDU (IX,IY,VY)
      IX=IY
      CALL RANDU (IX,IY,VX)
      VY=30+ (VY*16.67-8.33)
      VX=30+ (VX*16.67-8.33)
      IX=IY
      AWXX=X (6,J)*DTHTAX
      AWXY=-X (4,J)*DTHTAY
      AWYY=X (2,J)*DTHTAY
      AWYZ=X (6,J)*DTHTAZ
      AWZZ=-X (4,J)*DTHTAZ
      AWZX=-X (2,J)*DTHTAX
      AY=A (J)*THETA (2,J)
      THETA (1,JF1)=THETA (1,J)+DTHTAX
      THETA (2,JF1)=THETA (2,J)+DTHTAY
      THETA (3,JF1)=THETA (3,J)+DTHTAZ
      X (1,JF1)=X (1,J)+X (2,J)-VX+.5*(A (J)+AWXX+AWXY)
      X (2,JF1)=X (2,J)+A (J)+AWXX+AWXY
      X (3,JF1)=X (3,J)+X (4,J)+.5*(AY+AWYZ+AWYY)+VY
      X (4,JF1)=X (4,J)+AY+AWYZ+AWYY
      X (5,JF1)=X (5,J)+X (6,J)+.5*(AWZX+AWZZ)
      X (6,JF1)=X (6,J)+AWZX+AWZZ

C
C
      GAM IS THE NOISE INPUT FOR EACH ACCELEROMETER

      GAM (1) = EOG (1) + WG (1) * A (J)
      GAM (2) = EOG (2) + WG (2) * C
      GAM (3) = EOG (3)

C
C
      GENERATE THE BIAS TERM DUE TO RANDOM WALK

      CALL SNORM (IX,G,N)
      DO 50 JI=1,N
50      G (JI) = SIGW*G (JI)

C
C
C      PSI IS THE CHANGE IN THE ANGLE BETWEEN THE COMPUTED
      COORDINATE PLANE AND THE ACTUAL COORDINATE PLANE

      DO 51 JI=1,N
51      PSI (JI) = EO (JI) + G (JI)
      DELVX=VXO-VX
      DELVY=VYO-VY

C
C
      COMPUTE INERTIAL POSITION

      DTHTXM=DTHTAX+PSI (1)+DTHTAX*ZK (1)
      DTHTYM=DTHTAY+PSI (2)+DTHTAY*ZK (2)

```


DTHTZM=DTHTAZ+PSI(3)+DTHTAZ*ZK(3)

THE GAINS G1 AND G2 ARE THE KALMAN GAINS GENERATED AS
A FUNCTION OF TIME

G1=1.0-2.0/(TIMEX+1.0)
G2=1.0/(TIMEX+1.0)

THE COMMANDED HEADING CHANGE IS SUBTRACTED FROM THE
OBSERVED HEADING CHANGE

DLYJ=DTHTYM-DTHTAY
DLXJ=DTHTXM-DTHTAX

THE FILTER UPDATE EQUATIONS FOLLOW

THETAI(1,J)=THETAI(1,J)+G1*(DLXJ-OMEGAX)
OMEGAX=OMEGAX+G2*(DLXJ-OMEGAX)
THETAI(2,J)=THETAI(2,J)+G1*(DLYJ-OMEGAY)
OMEGAY=OMEGAY+G2*(DLYJ-OMEGAY)

THE FILTERED UPDATES ARE USED TO PREDICT THE NEXT
STATE IN THE NAVIGATOR

THETAI(1,JP1)=THETAI(1,J)+DTHTAX+OMEGAX
THETAI(2,JP1)=THETAI(2,J)+DTHTAY+OMEGAY
THETAI(3,JP1)=THETAI(3,J)+DTHTZM
TIMEX=TIMEX+1.0

SENSED ACCELEROMETER INPUTS IN THE BODY AXIS FRAME
IN THE X(BODY FRAME) DIRECTION

BETA1=A(J)-DEL VX+DEL VY*THETA(2,J)+GAM(1)

IN THE Y(BODY FRAME) DIRECTION

BETA2=DEL VX*THETA(2,J)+DEL VY+GAM(2)

IN THE VERTICAL(BODY FRAME) DIRECTION

BETA3=0.0

THE PURPOSE OF THIS SECTION IS TO GENERATE THE
INERTIAL ESTIMATES OF POSITION BASED ON A PURE
INERTIAL COMPUTATION

AWXXI IS THE ACCELERATIONS DUE TO HEADING CHANGE
AFFECTING THE X DIRECTION FROM THE ANGLE CHANGE THETA
X. SIMILARLY AWXYI IS THE ACCELERATIONS AFFECTING
THE X DIRECTION DUE TO THETAY

AWXXI=XI(6,J)*DTHTAX
AWXYI=-XI(4,J)*DTHTAY
AWYYI=XI(2,J)*DTHTAY
AWZII=XI(6,J)*DTHTZM
AWZZI=-XI(4,J)*DTHTZM
AWZXI=-XI(2,J)*DTHTAX

DTHTXM=0.0
DTHTYM=0.0
DTHTZM=0.0
DTHTAX=0.0
DTHTAY=0.0
DTHTAZ=0.0

SENSED ACCELERATIONS IN THE BODY FRAME ARE CONVERTED
TO THE INERTIAL FRAME.

YIX=BETA1-BETA2*THETAI(2,J)
YIY=BETA1*THETAI(2,J)+BETA2
IF(J.EQ.15) GO TO 48
IF(J.NE.80) GO TO 49


```

C
C
C      GENERATE THE RANDOM ERROR IN THE POSITION MEASUREMENT
48  CALL SNORM (IX,XPOS,1)
    XPOS=XPOS*SIGMIC
    CALL SNORM (IX,YPOS,1)
    YPOS=YPOS*SIGMIC
    XI (1,J)=XPOS+X (1,J)
    XI (3,J)=YPOS+X (3,J)
49  CONTINUE
    XI (6,J)=X (6,J)

C
C
C      COMPUTE THE INERTIAL ESTIMATES OF POSITION
    AND VELOCITY
    XI (1,JP1)=XI (1,J)+XI (2,J)+.5*(YIX+AWXXI+AWXYI)
    XI (2,JP1)=XI (2,J)+YIX+AWXXI+AWXYI
    XI (3,JP1)=XI (3,J)+XI (4,J)+.5*(YIY+AWYYI+AWYZI)
    XI (4,JP1)=XI (4,J)+YIY+AWYYI+AWYZI
    XI (5,JP1)=XI (5,J)+XI (6,J)+.5*(AWZZI+AWZXI)
    XI (6,JP1)=XI (6,J)+AWZZI+AWZXI
    VXO=VX
    VYO=VY
    IF (XI (1,JP1).GE.40000.) DTHTAX=.4538-THETA (1,JP1)
    IF (XI (5,JP1).LE.5000.) DTHTAX=-THETA (1,JP1)
    IF (DABS (XI (3,JP1)).LE.150.) GO TO 88
    REM=240000.-XI (1,J)
    IF (REM.LE..001) GO TO 88
    CONTRL=X (3,J)/REM
    DTHTAY=-CONTRL-THETA (2,JP1)
88  CONTINUE

C
C
C      THIS SECTION GENERATES THE REQUIRED STATISTICS
    DC 89 K=1,6
    ERR (K)=X (K,J)-XI (K,J)
    XMEAN (K,J)=XMEAN (K,J)+ERR (K)
    XBAR (K,J)=XBAR (K,J)+X (K,J)
    XEVAR (K,J)=XEVAR (K,J)+X (K,J)**2
    IF (ABS (ERR (K)).LE.0.001) GO TO 89
    XVAR (K,J)=XVAR (K,J)+ERR (K)**2
89  CONTINUE
    IF (XI (1,J+1).GT.240000.) GO TO 90
    GO TO 100
90  DO 91 I=1,6
91  XFIN (I)=X (I,J)
    XIFIN (1)=XI (1,J)
    XIFIN (3)=XI (3,J)
    XIFIN (5)=XI (5,J)
    GO TO 110
100 CONTINUE
    DC 101 I=1,6
101 XFIN (I)=X (I,150)
    XIFIN (1)=XI (1,150)
    XIFIN (3)=XI (3,150)
    XIFIN (5)=XI (5,150)

C
C
C      THIS SECTION COMPUTES THE FINAL VALUE STATISTICS
110 DO 120 J=1,6
120 XEFIN (J)=XBFIN (J)+XFIN (J)
    XIBFN (1)=XIBFN (1)+XIFIN (1)
    XIBFN (3)=XIBFN (3)+XIFIN (3)
    XIBFN (5)=XIBFN (5)+XIFIN (5)
    XERR1=XFIN (1)-XIFIN (1)
    XERR3=XFIN (3)-XIFIN (3)
    IF (XERR1.LE..001) GO TO 121
    XBFV (1)=XBFV (1)+XERR1**2
121 IF (XERR3.LE..001) GO TO 200
    XBFV (3)=XBFV (3)+XERR3**2
200 CONTINUE

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